Sequence Selection for Nonlinear Probabilistic Shaping: Toward a Practical Implementation

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Abstract *The performance and implementation of nonlinear probabilistic shaping is discussed. We propose a sequence selection scheme that avoids the interaction with forward error correction and achieves a gain of* 0.082 *bits/s/Hz.*

Introduction

Nowadays, probabilistic amplitude shaping (PAS) is acknowledged as an effective technique to improve performance and flexibility of an optical fiber network[1]–[3]. The success of PAS drove the research towards nonlinear shaping, a way to obtain additional gains in the nonlinear regime, possibly without increasing the complexity[4],[5]. Numerous attempts in this direction confirm that nonlinear shaping gain can be obtained in some specific scenarios[4],[6]–[11]. Furthermore, the sequence selection technique has been proposed to understand the potential and limitations of nonlinear shaping[12],[13]. In a nutshell, by selecting only *good* sequences for transmission and discarding all others, sequence selection provides an improved capacity lower bound in the nonlinear regime. In this work, we describe possible methods for the practical implementation of probabilistic shaping based on sequence selection, highlighting their capabilities and limitations.

System description

The simulation setup is as follows[14]. A dual polarization signal with 5×46.5 GBd channels spaced 50 GHz (root raised cosine with roll-off 0.05) and modulated with PAS-64QAM with rate $R = 9.2$ bits/4D propagates into a link made of 30 spans of 80 km single mode fiber ($\beta_2 = 21.7\,\mathrm{ps}^2/\mathrm{km}$, $\alpha = 0.2$ dB/km, $\gamma = 1.27\,\mathrm{W}^{-1}\mathrm{km}^{-1}$) with loss compensated by EDFAs with noise figure $N_F = 5$ dB. At the receiver, the central channel is demultiplexed, dispersion compensation is applied, followed by matched filtering, sampling at symbol time, and compensation of the mean phase rotation. The launch power is set to the optimal value $P = 1\,\mathrm{dBm}$.

The spectral efficiency (SE) theoretically achievable with sequence selection is evaluated by using the "averaged procedure" described in[13] with 20 realizations and sequences of length $n = 512$ 4D symbols. The proposed sequences are randomly drawn from a PAS scheme, considering either i.i.d. samples with a Maxwell–Boltzmann (MB) distribution (ideal scheme optimized for the linear regime)[15] or sphere shaping (SpSh) with block length 256 (best *conventional* PAS in the nonlinear regime)[16],[17], and a serial map of amplitudes to 4D symbols[17]. The selection metric is computed by using the enhanced split step Fourier method with 30 steps, and the best ηN_p sequences out of $N_p = 36864$ proposed sequences are selected, with $0 < \eta \leq 1$ being the acceptance rate.

Towards the implementation of sequence selection

To the best of our knowledge, the first attempt to a practical implementation of sequence selection is the list-encoding constant composition DM (CCDM) structure[18], which selects the best sequence—in terms of energy dispersion index (EDI)[10]—among those generated by appending some *flipping* bits to $\mathbf{b}$, the sequence of information bits, and using the CCDM. The list-encoding CCDM was later extended to account for different metrics and shaping techniques[19],[20]. The main drawback of this scheme, apart from complexity, is that the generation of the sequences is strictly related to the shaping strategy: a DM must be used, the sequence length n should be compatible with the shaping block length, and the sign bits are not considered in the selection.

However, this constraint can be overcome by splitting the modulation (which can include or not shaping) from the generation of different sequences, as in the bit scrambling (BS) sequence selection in the top left of Fig. 1[14]. For a given bit sequence $\mathbf{b}$, the BS generates N_t different test sequences by XOR with N_t different random-like

https://doi.org/10.1049/icp.2023.2626

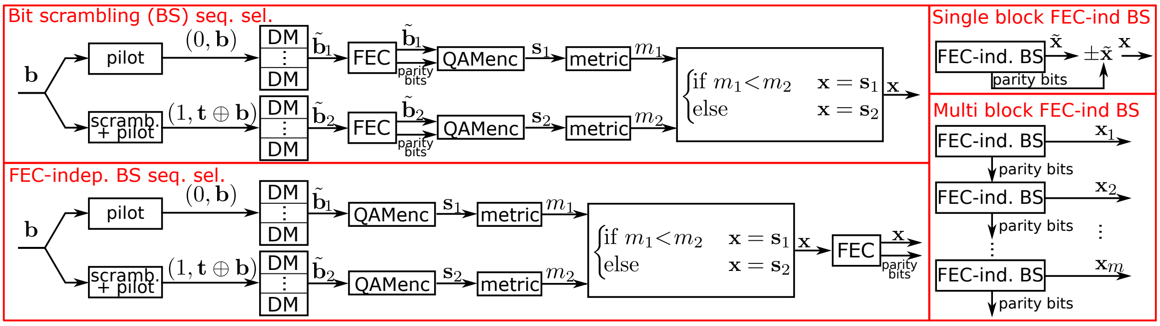

Fig. 1: Schemes for sequence selection with BS approach.

sequences, and adding $n_p = \lceil \log_2 N_t \rceil$ pilot bits. For each of these, (i) several instances of DM are used to produce shaped amplitudes (if shaping is used), (ii) FEC is applied, (iii) M-QAM encoder produces a sequence of n 4D symbols, encoding the FEC parity bits in the signs as in PAS, and (iv) a metric is evaluated. Next, the best sequence is used for transmission. At the received side, after conventional DSP, the detected symbols undergo FEC decoder and inverse DM, and the information bits are finally recovered thanks to pilot bits. The length of the sequences n and the DM block length are independent, since the output of many DMs can be concatenated to generate the sequences for selection. The main drawback of BS sequence selection is the complexity, since DM, FEC and metric evaluation should be performed for all N_t sequences. Furthermore, n should be compatible with the input length of the FEC.

The symbol interleaving (SI) sequence selection partially reduces the complexity of BS since it applies DM and FEC only once[14]. However, in this case, pilot symbols are used, which are not protected by the FEC and induce a larger loss w.r.t. BS.

The FEC-independent BS approach in the bottom left of Fig. 1 illustrates a simple way to solve the issues related to FEC in the BS scheme, moving the FEC encoder *after* sequence selection. However, in this case, the FEC parity bits need to be placed *somewhere*. Note that the BS approach reduces to the FEC-independent BS when the metric does not depend on the signs but only on the amplitudes, since the parity bits can be directly evaluated after selection and placed on the signs. The EDI, the windowed Kurtosis[4], the nonlinear phase noise[17], and the lowpass filtered symbol amplitude sequence (LSAS)[20] do not take into account the signs of the symbols and are compatible with this simple scheme.

To understand the potential and limitations of a metric that does not take into account the signs, Fig. 2(a) compares the SE achievable with ideal sequence selection as a function of the acceptance rate η ($N_t = 1/\eta$ on the top axis is useful for comparison with Fig. 2(b)) and two different shaping strategies: both amplitudes and signs (red); only amplitudes with unknown and unshaped uniform signs (blue). The proposed sequences are drawn from SpSh (solid) or have i.i.d. MB symbols (dashed). As expected, shaping also the signs provides the best performance. On the other hand, if the signs are not shaped and are unknown during the selection process, sequence selection provides only a little gain in the MB case—in practice, the same gain that SpSh without selection offers over MB (a little bit more, since ideal MB is not affected by the rate loss that affects any practical distribution matcher with finite block length). This result indicates that sequence selection is nearly useless if the signs are not accounted for in the selection process, since the same gain can be more easily achieved by optimizing the blocklength of a *conventional* PAS scheme.

Overall, the results obtained with sequence selection imply that the signs cannot be simply neglected during the selection process. To use the simpler FEC-independent BS sequence selection, one should use a sign-dependent sequence selection and find room for the FEC parity bits, without significantly altering the properties of the selected sequence. A simple solution is that of using the FEC-independent BS scheme in Fig. 1(b) with a sign-dependent metric, mapping the parity bits on some of the signs (hence altering only those signs). This means that sequence selection takes into account all the amplitudes and some of the signs, those which are not occupied by the parity bits. The ratio between the number of parity bits and signs bits is $\nu = (1-c)\log_2 M/2$, with c being the FEC rate; e.g., when $c = 4/5$ and $M = 64$, $\nu = 60\%$ of the signs are occupied by the parity bits and cannot be shaped. The performance of this method—referred to as single block FEC-independent BS (SB-BS) and sketched in the top right of Fig. 1—is compared with that of BS and SI

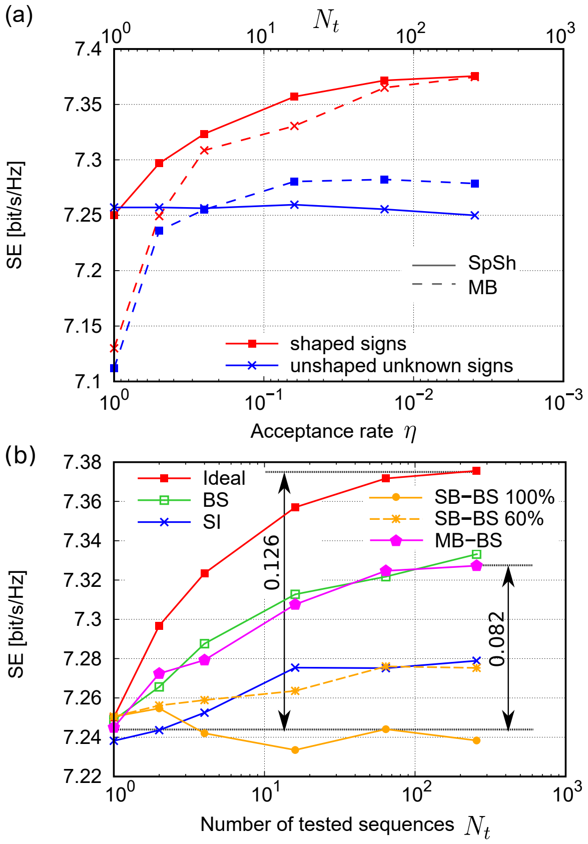

Fig. 2: SE with sequence selection with (a) ideal approaches and SpSh (solid) and MB (dashed) (b) with practical implementations with SpSh.

sequence selection in Fig. 2(b). When $\nu = 100\%$ (i.e., $c = 2/3$) all the signs are parity bits, and the SB-BS does not provide any improvement, as a consequence of the results in Fig. 2(a). When $\nu = 60\%$ (i.e., $c = 4/5$) a slight improvement is achieved since part (40%) of the sequence is correctly selected including the signs). In this case, the performance is approximately the same of the SI sequence selection.

Finally, the multi block FEC-independent BS (MB-BS) sequence selection concatenates different FEC-independent BS blocks, as sketched on the bottom right of Fig. 1. In a nutshell, the MB-BS scheme sends the parity bits to (part of) the signs of the symbols in the following block of FEC-independent BS selector, meaning that these bits are not shaped but are already known when the selection of the following block is performed. As shown in Fig. 2(b) MB-BS approaches the performance of BS, but significantly mitigates its complexity. The main disadvantage of the MB-BS sequence selection w.r.t. to the previous approaches is the latency, since encoding and decoding of different blocks should be performed in series (rather than in parallel) and the decoding can start only when the last block has arrived.

Conclusions

This work analyzed and compared different sequence selection schemes for nonlinear probabilistic shaping. Overall, the results show that (i) it is possible to devise simple and effective schemes for sequence selection, but (ii) this scheme must account for the signs of the symbols.

All the approaches discussed for sequence selection are affected by the complexity of the metric, which should be evaluated for all N_t sequences. The results in Fig. 2(a) showed that the signs should be included, since the metrics that depend only on the amplitudes (e.g., EDI, LSAS, Kurtosis, and NPN) offer a too little gain. In our case, we used the nonlinear interference estimated with a simplified channel model, whose complexity is still too high for a practical implementation. However, we believe that it is possible to devise a simpler sign-dependent metric for an accurate ranking of the sequences.

Finally, all the approaches discussed for BS sequence selection are affected, in principle, also by the complexity of the DM. This problem can be easily mastered using a DM with negligible complexity, as the hierarchical DM[21]. Furthermore, the BS approach can be modified to evaluate the DM only once, followed by a symbol interleaver to generate different sequences without affecting their distribution.

Acknowledgment

This work was partially supported by the European Union under the Italian National Recovery and Resilience Plan (NRRP) of NextGenerationEU, partnership on “Telecommunications of the Future” (PE00000001 - program “RESTART”).

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