



Persistence in firm growth: inference from conditional quantile transition matrices

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Abstract We introduce a new methodology to investigate the degree of persistence in firm growth dynamics, based on Conditional Quantile Transition Probability Matrices (CQTPMs) and exact inferential tests derived from two well-known mobility indexes. We apply the methodology to study manufacturing firms in the UK and four major European economies over the period 2010–2017. We find that CQTPMs display more persistence than under a fully independent firm growth process, albeit considerable turbulence and significant bouncing effects are detected. Exploiting the inferential statistics within a regression framework, we show that productivity, openness to trade, and business dynamism are the primary sources of firm growth persistence across sectors, while country-specific and time-specific factors play a second-order role.

Plain English Summary Use mobility indexes from CQTPMs to make precise inference about persistence in firm growth performance! This paper

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proposes a new method to measure the degree of persistence in the growth process of firms. Improving upon previous studies, we provide exact statistical inference and control for possible confounding factors. Application of our method to firm-level data on the UK and four major European economies, reveals a statistically significant, albeit small tendency of firms to repeat their past growth. However, we also observe considerable turbulence in the growth rates distribution over time. Persistence in growth is more strongly associated with sectoral characteristics such as productivity, business dynamism and trade openness, than with country-specific or time-specific factors. These findings pose interesting challenges to the theory. They also imply that policies addressed to foster growth in a specific group of firms are likely to have a volatile and transient effect. Our approach is flexible and suited for wide applicability in other domains of firm-level analysis.

Keywords Firm growth persistence · Transition matrices · Mobility indexes · Non-parametric statistics

JEL Classification C14 · D22 · L25

1 Introduction

To what degree are firm growth rates persistent? Does *success breeds success*, in the sense that currently

expanding firms show a higher probability of further expanding their market share, while those that are shrinking are destined to continue shrinking over time? Or, conversely, do industry dynamics unfold through mean-reverting or even random growth patterns, ultimately leading to instability in firm growth rates over time?

The answers to these questions have relevant implications for both theory and policy. From a policy perspective, studying firm growth persistence is important for understanding the extent to which financial support schemes or regulatory changes targeting firm growth, can actually promote durable economic growth, employment and value creation in sectors and countries. Growth policies have more likelihood of producing long-lasting effects — provided the right firms are targeted — if firm growth rates show a high persistence, implying that firms that are already growing tend to repeat their positive performance over time. Conversely, policies to sustain growth are likely to have only temporary effects, even when the correct firms are targeted, if firm growth exhibits low or even negative persistence. In this case, targeted firms that initially grow, are very likely to stop growing relatively quickly. At the same time, the empirical assessment of firm growth persistence is also important for anti-trust policy. In fact, although growth may not continue forever, for instance, due to a saturation of demand, strong persistence suggests a tendency toward the rapid emergence of firms with strong market power. That would call for timely policy interventions in those sectors in which market concentration is undesirable. Evidence of low or negative persistence instead suggests that industry dynamics follow a process that involves substantial market shares reshuffling and instability, thereby reducing the likelihood that strongly dominant firms will emerge and persist in a market.

From a theoretical viewpoint, studying firm growth persistence helps in comparing the empirical merit of alternative models of firm growth and firm-industry dynamics. Firstly, any sign of persistence, even low, would be at odds with models that describe the evolution of the firm size over time as the outcome of purely random, serially independent growth shocks (Gibrat, 1931; Geroski, 2000). Secondly, the degree of persistence observed in the data could help discriminate between models that assume convex vs. non-convex adjustment costs (Rothschild, 1971).

Convexities entail some degree of persistence in firm growth due to a smooth convergence toward optimal size, while non-convexities predict low or negative persistence due to lumpiness in investment and (S,s)-practices characterizing firm dynamics (see the review in Caballero (1999)). In addition, considerable persistence in growth patterns is implied by models such as those in the evolutionary tradition, which rationalize industry dynamics as stemming from the interaction between persistently superior vs. persistently inferior firms (Nelson and Winter, 1982; Silverberg et al., 1988; Dosi et al., 1995; Metcalfe, 1998; Dosi et al., 2000). In this family of models, in fact, persistent differences in relative firm-specific capabilities (e.g., in efficiency or innovation, due to cumulateness of knowledge, increasing returns, stability of routines and organizational structures, or path-dependence), lead to persistent differences in market outcomes, particularly in terms of profitability and growth. Notably, neoclassical equilibrium models of industry dynamics with firm heterogeneity, deliver similar predictions, although the core mechanisms differ across models, according to either passive learning (Jovanovic, 1982; Hopenhayn, 1992) or active learning (Ericson and Pakes, 1995).

The considerable information content that firm growth persistence carries for both theory and policy, has generated a large empirical literature, which we briefly review in Section 2. In summary, there are three types of empirical analyses. The vast majority of works study the average autocorrelation or autoregressive (AR) structure of firm growth rates, with inconclusive results, ranging from positive to negative to insignificant autocorrelations. More recent studies have extended the AR analysis by applying quantile regression (QREG) techniques. These report negative autocorrelations at both the bottom and the top quantiles of the growth rates distribution, but disagree as to whether the relative abundance of small-micro firms in these quantiles represents a convincing explanation for their results. Thirdly, a limited number of papers exploit transition probability matrices (TPMs) defined on growth rates quantiles (Quantile TPMs, QTPMs) to examine intra-distributional dynamics (Dopke and Weber, 2010; Capasso et al., 2013; Daunfeldt and Halvarsson, 2015; Mathew, 2017; Dosi et al., 2020). These QTPM studies tend to show that persistence is overall relatively low, as the majority of firms frequently move across the growth

rates quantiles over time. Also, they find peculiar patterns in the extreme quantiles. The firms in the top quantiles (i.e., the top-performers) and in the bottom quantiles (the under-performers), both show a higher probability of retaining their relative positioning over time than other firms, but they also undergo significant anti-persistent, bouncing effects.

This paper relates to this third stream of research that exploits TPMs and QTPMs in search of a more general characterization of growth persistence than the AR model. We improve upon previous works in two substantial ways, which we detail in Section 4. First, we introduce the Conditional Quantile TPMs (CQTPMs) to correctly assess the frequencies in QTPM cells, accounting for the dependence on additional confounding variables. We use this technique to remove any spurious persistence possibly arising from the well-documented relation between properties of firm growth QTPMs and firm size (Daunfeldt and Halvarsson, 2015; Capasso et al., 2013). Through the CQTPM approach, we can replace the *ex ante* defined firm size classes used in previous studies, with a conditional definition that adapts to the evolution of the size distribution inside each industrial sector. We also control for time-, country- and sector-specific effects, thus obtaining CQTPMs that are not affected by the spurious dependence possibly induced across firm growth quantiles by factors such as business cycle phases, demand dynamics, or technology patterns in individual sectors or countries.

Our second improvement, is the development of a framework to draw formal inferences regarding the overall degree of persistence in the intra-distributional dynamics described by transition matrices. Starting from the CQTPMs, we consider two mobility indexes—the Prais/Shorrocks index (Prais, 1955; Shorrocks, 1978) and the Bartholomew index (Bartholomew, 1982)—on which we can build a formal test for the null hypothesis of independent growth rates. Without a precise inferential analysis, the qualitative discussion of these indexes attempted in previous studies (Dopke and Weber, 2010; Dosi et al., 2020) is inconclusive. Drawing a precise inference is difficult, since the asymptotic properties of the elements of the CQTPMs and the associated mobility indexes depend on the joint density of the variables under study. We show that, under the null of independence, inferential analysis is in fact feasible through a relatively easy Monte Carlo exercise. The asymptotic standard

errors derived via the Monte Carlo analysis enable us to define a standardized, asymptotically normally distributed version of the mobility indexes. This forms the basis for a formal test of the null of independence.

In the empirical part of the paper, we apply CQTPMs and mobility indexes to examine persistence in sales growth dynamics at the aggregate economy and the sectoral level, exploiting a cross-country firm-level dataset. As we describe in Section 3, our data include a large sample of manufacturing firms active over the period 2010–2017 in the four major European economies (France, Germany, Italy, Spain) and the UK. In Section 5, we show that country-level CQTPMs are more persistent than under the null of fully independent growth rates. However, they also reveal a good deal of turbulence in the distributional dynamics. In particular, top-performers and under-performers are more persistent than other firms, and are also more likely to experience the bouncing effects reported in previous studies. On the other hand, our analysis of sectoral CQTPMs (by 2-digit sectors) reveals considerable variation in the extent of persistence across sectors, countries and time. Since we find that this variation is primarily driven by sector-specific effects, we then explore, in Section 6, the relation between sectoral standardized indexes and a set of sectoral characteristics. Dosi et al. (2020) made a similar attempt, correlating (unstandardized) mobility indexes with industry growth. In our case, we explore a wider set of industry-level variables, including sectoral characteristics that are commonly considered to be tightly linked to patterns of firm growth and firm-industry dynamics. We find that productivity, business dynamism and openness to international markets inversely relate to persistence. We discuss the implications of our study in Section 7, together with suggestions for future research.

2 Empirical literature on firm growth persistence

The empirical assessment of firm growth persistence has traditionally relied upon the estimation of autoregressive (AR) firm growth models, in panels of firms active in a given sector or country over time, possibly controlling for additional covariates (typically the initial firm size). This literature on the AR structure of firm growth rates is vast and the results are not easily comparable, as the studies differ in terms

of firm growth proxies, country, sector and the time period considered. A fair reading of these works is that they are far from delivering a consistent picture. Early studies, based on relatively small samples of large firms (see e.g., Ijiri et al. (1967); Kumar (1985); Dunne and Hughes (1994)), report a positive autocorrelation. Subsequent works did not confirm this finding in larger and more detailed samples. Positive autocorrelation was reported in Chesher (1979) for UK listed companies, in Wagner (1992) for the German manufacturing sector and in Bottazzi and Secchi (2003) for US manufacturing, while long-lasting autocorrelation (till the 7th lag) was found in Bottazzi et al. (2001) for the international pharmaceutical industry. Conversely, a number of works reported a negative serial correlation, e.g., in Boeri and Cramer (1992) for Germany, in Goddard and P (2002) for Japanese listed firms, and in Bottazzi et al. (2007) and Bottazzi et al. (2011) across Italian and French manufacturing firms, respectively. In addition, some studies found no serial correlation at all, such as Geroski and Mazzucato (2002) for the US automobile sector and Bottazzi et al. (2002) for Italian manufacturing industries, while other studies documented that the sign of the autoregressive coefficients varies with the lag considered, as in Coad (2007).

Most studies following the “AR approach” to firm growth persistence, estimate AR firm growth models either via standard estimators (OLS and panel) or LAD regression. They thus focus on the central tendency of the sample. Some studies however extended the AR analysis to also examine serial correlation along the quantiles of the growth rates distribution, by applying quantile regression (QREG) techniques (see, e.g., Lotti et al. (2003); Coad (2007); Coad and Hözl (2009); Capasso et al. (2013); Daunfeldt and Halvarsson (2015)). These studies tend to agree that growth rates autocorrelation (or anti-correlation) is generally weak, irrespective of the quantile considered. There are however some interesting variations. Growth rates serial correlation is very low or practically zero in the central quantiles, while negative autocorrelation is generally found across both low-performing firms in the bottom quantiles and high-growth firms in the top quantiles. Negative autocorrelation in the top quantiles is particularly relevant, as it speaks to the debate on the role of high-growth firms for long-run growth. In fact, this finding casts doubt that high-growth firms persist in their growth performance. This is in

line with previous evidence that high-growth firms are most often one-hit wonders rarely repeating their high-growth performance over time (Daunfeldt and Halvarsson, 2015), while the few persistent high-growth firms do not have clear advantages in terms of productivity and other key dimensions of performance (Bianchini et al., 2017; Moschella et al., 2019).¹

Concerning the explanation of the observed negative autocorrelation in the extreme quantiles, studies focused so far on the possible role played by firm size, with mixed evidence. The findings in Coad (2007) for French manufacturing firms, suggested that negative autocorrelation in the tails is due to small firms, especially in the case of small high-growth firms in the top quantiles. Coad and Hözl (2009) corroborated this conclusion in a comprehensive sample of Austrian service firms, showing in particular that negative autocorrelation is specific of very small, micro firms. In contrast, the estimated QREG-AR coefficients were significantly negative also for larger firms, in the near-population sample of Dutch firms examined in Capasso et al. (2013).

A major limitation of studies in the “AR approach” to firm growth persistence, irrespective of whether they examine central tendency or QREG estimators, lies in the implicit assumption that the growth rates of all firms follow the same parametric and linear process over time. This is a restrictive untested hypothesis, which does not necessarily hold true. In addition, the AR model describes the growth dynamics of each single firm independently of the dynamics of the other firms in the reference population, thus disregarding intra-distributional dynamics over time.

A more general characterization of firm growth persistence that addresses these limitations, is offered by the very few papers which analyze growth rates TPMs, in particular Quantile TPMs (QTPMs), thus providing evidence on the degree of mobility/stability in the entire growth rates distribution (Dopke and Weber, 2010; Capasso et al., 2013; Daunfeldt and Halvarsson, 2015; Mathew, 2017; Dosi et al.,

¹ See also Erhardt (2021), showing that the extent of high-growth persistence observed empirically, is highly sensitive to the alternative measures of high-growth persistence used in the literature.

2020). A comparison of results across these studies is complicated by differences in samples, levels of analysis (country vs. sector), definitions of QTPM states (quartiles, deciles or percentiles of growth rates) and lengths of transition (usually 1 year, but in some cases longer, 3-to-5 year transitions). A few common findings emerge, however. First, the main diagonal elements of the estimated QTPMs are typically far below 1, meaning that the vast majority of firms do not keep their relative positioning over time. This is qualitatively interpreted as evidence of a strong deviation from a fully persistent process. Second, there are persistent out-performers and persistent under-performers in the top and bottom quantiles. Third, anti-persistent dynamics are often found in the off-diagonal cells, particularly in extreme quantiles, as indeed firms in the extreme quantiles in the initial period have a relatively high probability of ending-up in the opposite extreme quantiles. Bouncing effects of this type are more apparent in studies that take a comparatively more fine-grained definition of quantile-states (deciles or percentiles of growth), such as Dopke and Weber (2010) on German non-financial firms, Capasso et al. (2013) on the Dutch manufacturing, and Daunfeldt and Halvarsson (2015) on Swedish firms. The evidence is more nuanced in Mathew (2017) and Dosi et al. (2020), who examine mobility across growth rates quartiles, for Indian and US-COMPUSTAT firms, respectively. Apart from these few common findings, the QTPMs reported in the studies show considerable variability according to a number of factors, in particular firm size (Capasso et al., 2013; Daunfeldt and Halvarsson, 2015), sector of activity (Mathew, 2017; Dosi et al., 2020) and also time and business cycle (Dopke and Weber, 2010).

A key weakness of all these studies is that the discussion of QTPMs remains largely qualitative. There is no systematic attempt to provide a formal inferential analysis of the properties of the matrices, not even in the papers that introduced the mobility indexes also use in the present study (Dopke and Weber, 2010; Dosi et al., 2020). In general, the authors compare values in the QTPM cells and mobility indexes obtained in different sub-samples, interpreting their relative values as an indication of persistence. However, without assigning an accurate confidence interval to the

computed statistics, these qualitative comparisons are seldom informative.

3 Data and descriptive evidence on firm growth rates

The empirical analysis of this paper exploits the ORBIS database maintained by Bureau Van Dijk, which is a widely used source of information on financial statements and other firm characteristics, covering over 200 million firms across the globe. Although the limitations are well known, especially in terms of the under-representation of micro firms (below 10 employees), it constitutes the best available source for cross-country analysis (Kalemlı-Ozcan et al., 2015; Bajgar et al., 2020). For this work, we have access to data for France, Germany, Italy, Spain and the UK, over the period 2010–2017. We focus on manufacturing firms, classified according to their sector of primary activity at the 2-digit level (NACE Rev.2).²

We define the growth of firm i in year t as the log-difference

$$g_{i,t} = s_{i,t} - s_{i,t-1} \quad (1)$$

where

$$s_{i,t} = \log(S_{i,t}) - \frac{1}{N} \sum_i^N \log(S_{i,t}) \quad (2)$$

is the logarithm of annual revenues $S_{i,t}$ normalized by removing the average annual revenues computed over all the N firms active in the same (2-digit) sector as firm i . This normalization is often used in firm growth empirics to account for common factors affecting the size of all firms in the same sector. It implies that g

² ORBIS reports 4-digit industries. However, as shown in Appendix 1 in the online material, with a finer sectoral disaggregation than the 2-digit level (at 3 or 4-digit), the number of firms is too small for sensitive statistical analysis. In fact, computation of CQTPMs is heavily data demanding. Notice also that we do not consider NACE 12 (Manufacture of tobacco products), NACE 15 (Manufacture of leather and related products), and NACE 19 (Manufacture of coke and refined petroleum products) since the number of firms in these sectors is already too small (less than 20 firms) at the 2-digit level, in all countries.

Table 1 # of firms available to compute growth rates

| Year | France | Germany | Italy | Spain | UK |
|-------------|--------|---------|---------|--------|--------|
| [2010/2011] | 50,478 | 15,090 | 80,559 | 59,669 | 10,077 |
| [2011/2012] | 52,671 | 19,009 | 83,505 | 59,817 | 10,121 |
| [2012/2013] | 53,400 | 33,964 | 86,676 | 59,604 | 10,360 |
| [2013/2014] | 44,794 | 36,272 | 90,796 | 60,959 | 10,530 |
| [2014/2015] | 39,021 | 32,687 | 102,892 | 62,284 | 10,551 |
| [2015/2016] | 30,882 | 32,506 | 112,056 | 63,709 | 10,478 |
| [2016/2017] | 25,888 | 28,936 | 127,413 | 62,605 | 10,736 |

Table 2 AR persistence

| | Year | France | Germany | Italy | Spain | UK |
|--|-------------|---------------------|---------------------|---------------------|----------------------|----------------------|
| LAD estimates of the AR coefficient β in Eq. 3. Bootstrap standard errors in parentheses. Asterisks denote significance levels: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$ | [2011/2012] | 0.003 (0.003) | 0.003 (0.005) | 0.019*** (0.004) | 0.027*** (0.006) | -0.026** (0.013) |
| | [2012/2013] | 0.002 (0.003) | 0.000 (0.001) | 0.007** (0.003) | 0.011** (0.006) | -0.052*** (0.016) |
| | [2013/2014] | 0.002 (0.004) | 0.006** (0.003) | 0.013*** (0.003) | -0.020*** (0.005) | 0.025** (0.011) |
| | [2014/2015] | 0.018*** (0.007) | 0.011*** (0.004) | 0.018*** (0.003) | 0.008* (0.004) | -0.018 (0.011) |
| | [2015/2016] | 0.000 (0.006) | 0.004 (0.003) | 0.001 (0.002) | 0.008* (0.004) | -0.030* (0.016) |
| | [2016/2017] | 0.006 (0.006) | 0.010* (0.005) | 0.001 (0.002) | -0.001 (0.005) | 0.007 (0.010) |
| | All years | 0.004** (0.002) | 0.005*** (0.002) | 0.009*** (0.001) | 0.006*** (0.002) | -0.010** (0.004) |

measures the growth of relative size, thus capturing market shares dynamics.

We take annual sales as the empirical proxy of firm size, but other proxies are possible and have been used in firm growth empirics, such as employment or tangible assets. The latter two proxies describe the input side of the growth processes and are suited to capture the growth in production capacity, relating to labor and investment dynamics. Taking sales as the proxy of size, the growth rates measured adhere more closely to the notion of growth that theories of firm-industry dynamics typically consider, i.e., the ability to succeed in the output market.

Table 1 shows the number of firms for which we can compute 1-year growth rates, as they report non-missing values of sales over two consecutive years. As ORBIS is an unbalanced panel, the observations vary by year.

3.1 Autoregressive analysis of growth persistence

An AR model of firm growth of the form

$$g_{i,t} = \beta g_{i,t-1} + \epsilon_{i,t} \quad (3)$$

or variation thereof (e.g., including further lags on the right hand side) is the most common empirical approach in the literature to assess persistence in growth rates and is therefore a useful benchmark to start with. An AR(1) coefficient β statistically equal to zero indicates no persistence, while a significantly

positive (negative) estimated β provides evidence of serially autocorrelated (anti-correlated) growth episodes over time. Ideally, β should be estimated separately for each firm, but the time series dimension of standard firm-level datasets is usually too short to allow for a reliable firm-by-firm estimation.³ With 7 years available in our data for computing firm growth rates, we follow the common practice of pooling firm-year observations, implying that the estimated β captures the average growth rates autocorrelation in the sample.

Table 2 reports country estimates of the AR specification in Eq. 3, obtained through the LAD estimator, which is robust for non-normalities in

³ See Dosi et al. (2020) for a notable exception, exploiting long firm-level time series from US-COMPUSTAT data.

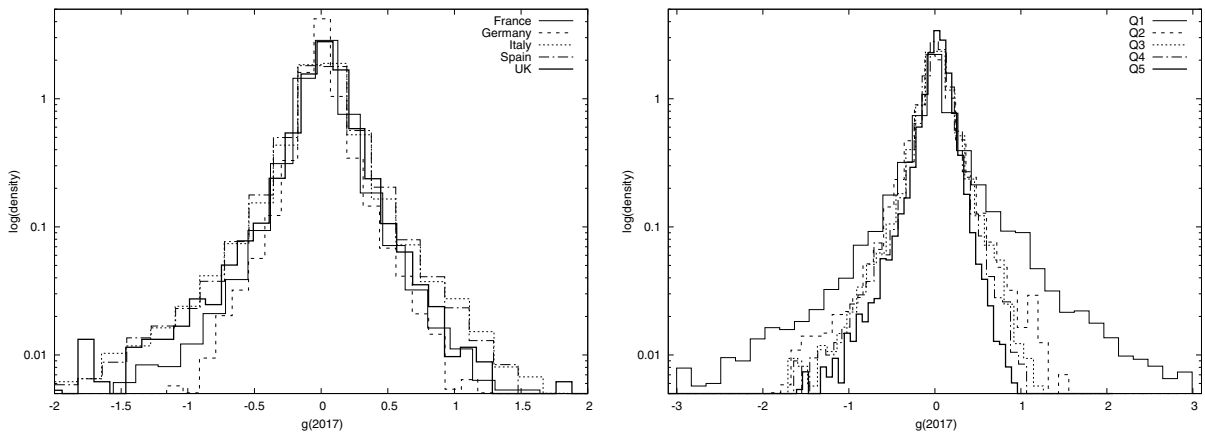


Fig. 1 *Left*: Empirical density of firm (sales) growth rates, estimates for aggregate manufacturing in the year 2017, by country. *Right*: Empirical density of firm growth rates conditional on firm size, estimates for aggregate manufacturing in

Italy, reporting sales growth rates in 2017 by quintiles of firm size (sales) in 2016. Comparable results are obtained in other years, in all countries

the distribution of the dependent variable. This estimator is appropriate since our data replicate the stylized fact that firm growth rates exhibit a fat-tailed, tent-shape behavior (see Fig. 1, left panel), robustly documented in the literature across countries, levels of sectoral aggregation and time periods (see, e.g., (Stanley et al., 1996; Amaral et al., 1997; Bottazzi and Secchi, 2003; Bottazzi and Secchi, 2006a; Coad, 2009; Bottazzi et al., 2011; Bottazzi et al., 2014)).

When pooling data over the available years (last line in Table 2), coefficient estimates are close to zero, despite statistically significant, implying a very low persistence. Separate estimates by year confirm that AR persistence is generally low. In fact, despite some variability over time and across countries, coefficient estimates are not statistically different from zero in most cases, or when statistically significant they range between -0.052 and 0.027.

3.2 Relation between growth rates and firm size

A general problem affecting the estimation of AR models is the heteroskedastic nature of growth rates, referring to the dependence between firm size and the distribution of firm growth rates (Stanley et al., 1996; Amaral et al., 1997; Bottazzi et al., 2001; Bottazzi and Secchi, 2003; Bottazzi and Secchi, 2006b; Calvino et al., 2018).

In fact, size-growth dependencies are present and strong in our sample. As illustrated in Fig. 1 (right panel) taking data on Italian firms between 2017 and 2016, the empirical density of sales growth rates changes across classes of initial firm size, identified here by quintiles of the initial sales distribution. Specifically, maximum likelihood estimates of shape and scale parameters of the asymmetric exponential power (AEP) distribution, in Table 3, reveal that smaller firms show a higher

Table 3 Growth rates distribution conditional on size — AEP parameters

| | Size quintiles | | | | |
|-------|------------------|------------------|------------------|------------------|------------------|
| | Q1 | Q2 | Q3 | Q4 | Q5 |
| a_l | 0.210 (0.003) | 0.180 (0.002) | 0.155 (0.002) | 0.147 (0.002) | 0.124 (0.002) |
| a_r | 0.239 (0.004) | 0.161 (0.002) | 0.147 (0.002) | 0.140 (0.001) | 0.125 (0.001) |
| b_l | 0.410 (0.005) | 0.611 (0.008) | 0.601 (0.008) | 0.603 (0.007) | 0.588 (0.007) |
| b_r | 0.375 (0.004) | 0.692 (0.009) | 0.774 (0.011) | 0.914 (0.013) | 1.084 (0.016) |

Scale (a_l and a_r) and shape (b_l and b_r) parameters of AEP estimates of sales growth rates distribution of Italian manufacturing firms in 2017, by quintiles of firm size (sales) in 2016. Standard errors in parenthesis. Comparable results are obtained in other years, in all countries

average growth and larger growth variance.⁴ We found consistent results also in the other years and countries included in our data, in line with previous findings in the literature.

One possible strategy to address the size-growth dependence within the AR approach, is to extend Eq. 3 to include an explicit scaling function accounting for heteroskedastic shocks (Bottazzi et al., 2007). In this paper we propose a different solution, which accounts for the size-growth dependence within our overall goal of studying firm growth persistence through transition matrices, allowing for a more general characterization of persistence than in a parametric AR structure.

4 Methodology

This paper improves upon previous uses of the Quantile TPMs (QTPMs) and related mobility indexes for the empirical analysis of persistence in firm growth rates. Compared to standard TPMs, QTPMs have a clear advantage. Standard TPMs rely upon pre-defined partitions of the support of the variables used to define rows and columns of the matrix. This can be reasonable for certain kind of data for which a natural or formal partition is commonly accepted but it is hardly justified when dealing with over time changes in the growth rates distribution. QTPMs are more robust in this respect, since they do not depend on the specific shape of the marginal distributions of the variables considered. The partitioning of the support of the variables is based on quantile functions, and therefore it is insensitive to any invertible monotonic transformation applied to the variables themselves. In the context of firm growth analysis, this means that the matrices obtained, for instance, for different sectors or countries, can be directly compared even if the growth rates distributions are, in those cases, different.

Despite the characterization of probabilistic dependence provided by QTPMs is far more general than the restrictive parametric assumption implicit in

AR models, there are two inherent difficulties in the application of QTPMs to firm dynamics. First, the direct application of the tool to a bivariate distribution (such as the bivariate distribution of two consecutive growth rates, g_t and g_{t-1}) might be affected by the dependence of the two considered variables on other variables. To overcome this difficulty, in Section 4.1 we introduce the Conditional Quantile TPM (CQTPM). Second, inference with QTPMs is more difficult than with standard TPMs. Analogously to the elements of a standard TPM, the elements of the empirical QTPM are consistent, efficient and asymptotically normally distributed estimators of the corresponding “true” elements, which could be obtained under complete knowledge of the underlying distributions. However, the asymptotic variance-covariance structure of matrix elements of a QTPM is more complicated than in the case of a standard TPM. In Section 4.2 we show how the choice of an appropriate null, that is the null of independence, helps solving this difficulty and makes the inferential analysis based on mobility indexes relatively simple.

4.1 Conditional Quantile TPMs

To see how the CQTPMs work, let us start with the definition of its unconditional version, the QTPM. Assume to have a sample of N paired observations $S = \{(x_n, y_n)\}$ with $n = 1, \dots, N$. The first element x_n is often associated with the *initial state* and the second element y_n with the *final state* of some observed variable. Let \hat{F}_x and \hat{F}_y be the empirical distribution of the values $\{x_n\}$ and $\{y_n\}$ respectively. In other terms, \hat{F}_x and \hat{F}_y are the marginals of the joint empirical distribution $\hat{F}(x, y)$, obtained from the sample S . The respective empirical quantile functions are defined as $\hat{Q}_x(u) = \inf\{x \mid \hat{F}_x(x) \geq u\}$ and $\hat{Q}_y(u) = \inf\{y \mid \hat{F}_y(y) \geq u\}$ for $u \in [0, 1]$.⁵ Given a partition of the interval $[0, 1]$ in Q equispaced intervals of size $1/Q$, consider the quantities $\hat{x}_i = \hat{Q}_x(i/Q)$ and $\hat{y}_i = \hat{Q}_y(i/Q)$ for $i = 0, 1, \dots, Q$. In particular, $\hat{x}_0 = \hat{y}_0 = -\infty$. Then, the QTPM matrix $\hat{P}(S)$ associated to the sample S is a $Q \times Q$ matrix defined as

⁴ The AEP distribution introduced in Bottazzi and Secchi (2011) is widely used in firm growth literature to assess asymmetries and fat-tails in growth rates. Left and right shape parameters (b_l and b_r) smaller than two correspond to tails fatter than a Gaussian. The left and right scale parameter (a_l and a_r) capture the width of the support below and above the modal value.

⁵ The quantile function is theoretically defined as the inverse of the distribution function. A more elaborate definition is required when the empirical distribution function is considered because the latter is in general not invertible.

$$\hat{P}_{i,j}(S) = \frac{\sum_{n=1}^N I\{\hat{x}_{i-1} < x_n \leq \hat{x}_i, \hat{y}_{j-1} < y_n \leq \hat{y}_j\}}{\sum_{n=1}^N I\{\hat{x}_{i-1} < x_n \leq \hat{x}_i\}}, \tag{4}$$

$i, j = 1, \dots, Q,$

where $I\{\cdot\}$ is the indicator function, taking value 1 if its argument is true, and 0 otherwise. The (i, j) element of the matrix \hat{P} contains the number of paired observations whose first component is between \hat{x}_{i-1} and \hat{x}_i (included) and whose second component is between \hat{y}_{j-1} and \hat{y}_j (included), divided by the number of observations whose first component is between \hat{x}_{i-1} and \hat{x}_i (included), irrespective of the value of the second component. Since the partition of the interval $[0, 1]$ is equispaced, the denominator in Eq. 4 is approximately equal to N/Q .⁶ Because the sample QTPM is a consistent and asymptotically efficient estimator of the true QTPM (see, for instance, Section 3.2.2 in (Formby et al., 2004)), if the observations are drawn from a joint distribution $F(x, y)$ with marginals F_x and F_y , then, for $N \rightarrow \infty$, one has

$$\begin{aligned} \hat{P}_{i,j}(S)/Q \rightarrow & F(F_x^{-1}(i/Q), F_y^{-1}(j/Q)) + F(F_x^{-1}((i-1)/Q), F_y^{-1}(j-1)/Q)) \\ & - F(F_x^{-1}(i/Q), F_y^{-1}(j-1)/Q)) - F(F_x^{-1}(i-1)/Q, F_y^{-1}(j/Q)). \end{aligned} \tag{5}$$

The matrix $\hat{P}(S)$ is bi-stochastic, that is the sum of the elements of each row and each column is equal to 1: $\sum_{j=1}^Q \hat{P}_{i,j} = \sum_{j=1}^Q \hat{P}_{i,j} = 1$ for any i, j .

The QTPM does not contain more information than the joint empirical distribution $\hat{F}(x, y)$, but it is convenient in highlighting the existence of dependence between the two components x and y of the paired observations. If larger values of x are more often paired with larger values of y , then the entries of the matrix P laying along or near the main diagonal will show relatively larger values. Conversely, if larger values of x are more likely paired with smaller values of y , then the elements away from the diagonal will have the larger values.⁷ Differently from standard regression models, which establish a specific functional relation between the variables x and y (in our context, the linear AR model relating g_t with g_{t-1}), the way QTPMs capture probabilistic dependence is

not constrained by a specific parametrization, and can easily account for the presence of non-linear effects.

There is however an important issue that may bias the interpretation of the observed QTPMs, potentially preventing that they carry precise information about the probabilistic dependence between the two considered variables. If there existed a third variable z on which the realizations of both x and y depend, then spurious patterns would appear in the QTPM cells due to the clustering of observations of x and y around the realizations of the third variable. To keep the parallel with regression analysis, this is similar to the bias that would arise due to omitting z in a regression between x and y .⁸

In the context of this paper, where the focus is on dependence in firm growth rates over time (that is, x is g_{t-1} and y is g_t), just computing the QTPM as done in previous firm growth studies, would assume that the probabilities in the QTPM cells correctly reflect the joint distribution of the two states, disregarding that firm growth may depend on other variables. That would ignore, for instance, the size-growth dependencies discussed in Section 3.2. But, in fact, any other omitted variable that correlates with firm growth — not only firm size — might create a spurious tendency toward overpopulating main diagonal entries of the matrix, resulting into an overestimation of persistence.

The conditional version of the QTPM, the CQTPM, exactly enables to account for “variable dependence” in QTPM analysis. Assume to augment the sample of ordered couples (x, y) under investigation with the observations on a third variable z potentially related to the first two. The sample S is now made of N triplets $S = \{(x_n, y_n, z_n)\}$ with $n = 1, \dots, N$. Then, in order to condition upon z , one can simply

⁶ Deviations are due to the sample size not being perfectly divisible by Q .

⁷ Larger and smaller must not to be intended in absolute terms, but rather relative to the values observed in the sample itself.

⁸ Dependence on a third variable could also spoil the interpretation of the QTPM as the transition matrix of a Markov process between states defined by values of x and y , and thus its use to obtain an invariant measure of the process. In fact, in a Markov process the initial state contains all information necessary to determine the distribution of the final states. This would hold in the case of QTPMs only if the first observation were a sufficient statistic for the realizations of the second, which is in general not guaranteed to be true. See, for instance, the exercise performed over productivity levels in Bartelsman and Dhrymes (1998) — that the authors rightly acknowledge as inconclusive — or the estimate of the Markov process driving the “growth regimes” in Dopke and Weber (2010).

split the sample according to the values of z itself and examine the QTPMs between x and y within each sub-sample defined according to the values of z . If z is a discrete variable, the split procedure is obvious as one can simply build a different sub-sample collecting observations on x and y for each different discrete value of z . In this case, the QTPMs computed according to Eq. 4 within each sub-sample, are themselves the CQTPMs, since they are by construction conditional upon the realization of z . Instead, if z is a continuous variable, one can resort to the quantile definition. Labeling as \hat{Q}_z the marginal empirical quantile function associated to z , the sample S can be split into L equipopulated sub-samples S_l defined as

$$S_l = \{(x_n, y_n) \mid \hat{z}_{l-1} < z_n \leq \hat{z}_l\}, \quad (6)$$

by setting $\hat{z}_l = \hat{Q}_z(l/L)$, with $l = 0, \dots, L$. Then, a CQTPM $\hat{P}(S_l|z)$ is build for each sub-sample S_l applying Eq. 4 above. The rationale is that when the third component z_n is constrained in a limited range of values, its effect on the first two variables x_n and y_n does not change significantly and thus can be neglected when building the transition matrix. The support of observations (x_n, y_n) is in general different for different sub-samples S_l , so that the different matrices $\hat{P}(S_l|z)$ are built using different empirical quantile functions \hat{F}_x and \hat{F}_y .

Once the CQTPMs relative to the different sub-samples are computed (one CQTPM for each discrete value of z or one for each quantile-based sub-sample S_l), they can be examined separately to assess properties of the relation between x and y which are now free of bias due to dependence on z . For instance, to identify properties that are robust vis-a-vis properties that vary across different values of z . Alternatively, it might be convenient to combine the CQTPMs computed over the different sub-samples, to obtain an “aggregate” CQTPM that describes the (now unbiased) relation between x and y . Upon checking for homogeneity, the aggregate CQTPM can be computed by averaging, as $\hat{P}(S|z) = \sum_{l=1}^L \hat{P}(S_l)/L$. One reason for averaging is increasing the size of the sample. This will be relevant in the inference analysis of the following sections.

The entire procedure to compute CQTPMs, can be generalized to the case where there are several variable $\{z_1, \dots, z_K\}$ to be controlled for. One simply needs to (i) split the original sample on x and y

in equipopulated bins with respect to the values of the K (discrete or continuous) omitted variables; (ii) compute the CQTPM relative to x and y separately on each sub-sample; and (iii), if useful, average across sub-samples to obtain the aggregate CQTPM. However, one should be parsimonious about the number of variables to condition upon, as the observations in the sub-samples may decrease rapidly with the number of conditioning variables. This is the price to be paid in exchange for the more flexible characterization of dependence that CQTPMs offer compared to a standard regression model.

4.2 Inference through mobility indexes

While transition matrices (in general, not only QTPMs or CQTPMs) provide a rich non-parametric description of the dependence between two variables, just looking at the numbers in specific cells or comparing cell values across matrices, might not be particularly informative. The identification of interesting patterns by visual inspection is not trivial when the matrices are large or there are many matrices to be compared. In addition, there are no guarantees that the supposedly identified patterns are statistically significant.

A number of so-called mobility indexes have been proposed in the literature to summarize the properties of or to extract specific information from QTPMs. Starting from the observed couples (x, y) , these indexes capture the tendency that the relative position of the realized value y in the empirical distribution of values $\{y_n\}$, is similar to the relative position occupied by the realization of initial variable x in the empirical distribution of values $\{x_n\}$.

We consider two indexes originating from studies of income distribution and recently imported in studies of firm growth persistence. The first is the Shorrocks or Prais/Shorrocks index (Prais, 1955; Shorrocks, 1978). It is defined as

$$I_s(P) = \frac{Q - \text{tr}(P)}{Q - 1}, \quad (7)$$

where P is the QTPM (or the CQTPM), Q is the number of quantiles and $\text{tr}(P)$ denotes the trace of the matrix. The second index we consider is the Bartholomew index (Bartholomew, 1982)

$$I_b(P) = \frac{1}{Q-1} \sum_{i=1}^Q \sum_{j=1}^Q \frac{n_i}{n} P_{ij} |i-j|, \quad (8)$$

where i and j indicate the initial and final quantiles identifying an entry of the QTPM (or the CQTPM), respectively, and n_i/n is the number of observations in the initial quantile i over the total number of observations, thus approximately equal to $1/Q$.

The two measures offer complementary yet different characterization of the degree of mobility/persistence in a matrix. The Shorrocks index I_s conveys information about the probability to remain in or move out from the initial quantile, by just considering the frequencies in the main diagonal of the matrix. If there is no mobility, that is when all observations y_n are in the same quantile of their respective x_n , the value of the index is zero. Then, the index increases with mobility: the more observations are characterized by a change in the relative order of x and y variables, the larger the index. The maximum value is equal to $Q/(Q-1)$, in the case where all the y_n occupy a different quantile than the respective x_n .

The Bartholomew index provides a broader account of all the movements occurring within the matrix, beyond the main diagonal. As the Shorrocks index, the I_b index also equals zero under no mobility, when all observations y_n remain in the same quantile of their respective x_n . However, in the case there is some mobility, all the matrix entries (even those out of the main diagonal) contribute to the value of the Bartholomew index, with a weight $|i-j|$ that increases with the distance between the initial and final quantiles. In this respect, the Bartholomew index is more apt than the Shorrocks' index to account for the anti-persistent, bouncing effects across distant quantiles that have been observed in previous qualitative studies of firm growth QTPMs. A firm jumping — say — from the first to the tenth decile, contributes to the Shorrocks index in the same way a firm moving from the first to the third decile does. The Bartholomew index gives different weights to these two movements, giving more weight to the longer jump.

In order to statistically compare the indexes computed over different matrices, or to draw inference about whether the index values obtained for a given matrix differ statistically from a given benchmark, one needs to assign a standard error to them. Theoretically, the indexes are statistics defined over the

entries of the matrix. Thus, their asymptotic properties derive from the asymptotic properties of the latter. In case the matrix is a QTPM, the asymptotic variance-covariance structure of the matrix entries is more complicated than in the case of a standard TPM. There are two reasons for this. First, while in a standard TPM the boundaries of the cells are fixed, in a QTPM they are themselves estimated from the empirical quantile function of the variables and, as such, subject to noise (see Section 4.1 and Formby et al. (2004), p. 189). Second, in a QTPM the number of observations in each row and column is constrained to be exactly N/Q . The increased complexity of the variance-covariance structure of the matrix elements induced by these two effects, implies that simple approaches as the one presented in Schluter (1998) are not suited. In particular, the delta method analysis in Formby et al. (2004) shows that the asymptotic variance-covariance structure of the QTPM elements depends on the joint probability density of the underlying variables. Thus, in order to derive the asymptotic behavior of the QTPM elements and of the associated mobility indexes, an estimate of the underlying joint density is needed. In many situations, this requirement makes the direct use of the asymptotic expression excessively cumbersome. A viable alternative could be to apply bootstrap techniques, along the lines suggested in Biewen (2002) and Richey and Rosburg (2018) for the case of mobility indexes derived from standard TPMs. In the case of QTPMs the bootstrap approach would be even more recommendable than in the case of standard TPMs. However, when conditional QTPMs are considered — as in this paper — the bootstrap procedure is complicated by the need to find an appropriate stratification of the sample with respect to the omitted/control variable.

In our case, a simpler approach is possible. The key observation is that our inferential analysis involves comparing the empirical indexes with the (asymptotic) distribution they have under the null of independent growth rates. In fact, inspired by the classical Gibrat's model of size-growth dynamics, independence of firm growth rates over time is the natural benchmark in our context. It has been already used to gauge qualitative considerations about firm growth QTPM entries in Capasso et al. (2013) and Dosi et al. (2020).

In general terms, the null of independence implies that, starting from any initial quantile, there is the same probability to end up in any one of the final quantiles. This corresponds to a theoretical matrix with $1/Q$ in all the entries, with Q the number of quantiles considered. Under this particular null, the derivation of the properties of the indexes greatly simplifies. First, it's easy to derive that, under the null, the expected values of the indexes computed empirically, \hat{I}_s and \hat{I}_b , read

$$\mathbb{E}[\hat{I}_s] = 1 \text{ and } \mathbb{E}[\hat{I}_b] = \frac{Q+1}{3Q}. \quad (9)$$

Second, when the variables are independent (i.e., under the null), the invariance of the QTPM under invertible monotonic transformation of the underlying random variables, implies that the asymptotic behavior of the variance-covariance structure of the QTPM elements becomes distribution independent. It can therefore be derived via a Monte Carlo approach.

Specifically, since the elements of the QTPM are consistent, efficient and asymptotically normal, then the considered indexes, being smooth functions of these quantities, are themselves consistent, efficient and asymptotically normal. Therefore, under the null of independence, with sample size N going to infinity, the indexes \hat{I}_s and \hat{I}_b are normally distributed with mean given in Eq. 9 and variance

$$\mathbb{V}_N[\hat{I}_x] \sim \frac{C_x(Q)}{N}, \quad x = s, b. \quad (10)$$

The asymptotic coefficients $C_s(Q)$ and $C_b(Q)$ only depend on the number Q of quantiles considered and can be therefore computed via Monte Carlo simulations with any distribution. Since in the paper we will perform the empirical analysis using deciles, we are interested in the case $Q=10$. Accordingly, we perform the following exercise. For a given sample size N , we randomly generate R independent samples of N couples of independent observations drawn from a given distribution. On each sample, we compute the QTPM with $Q=10$, and the Shorrocks and Bartholomew indexes associated to this matrix. We end up with R independent realizations for each index, $\hat{I}_x(r;N)$, with $r = 1, \dots, R$ and $x = s, b$. Then, we compute their mean and variance

$$\mathbb{E}_N[\hat{I}_x] = \frac{1}{R} \sum_{r=1}^R \hat{I}_x(r;N), \quad \mathbb{V}_N[\hat{I}_x] = \frac{1}{R-1} \sum_{r=1}^R (\hat{I}_x(r;N) - \mathbb{E}_N[\hat{I}_x])^2, \quad x = s, b.$$

When N goes to infinity, $\mathbb{E}_N[\hat{I}_x]$ goes to the value $\mathbb{E}[\hat{I}_x]$ reported in Eq. 9. Concerning the behavior of the variance, we report in Fig. 2 the quantities $N \mathbb{V}_N[\hat{I}_s]$ (left panel) and $N \mathbb{V}_N[\hat{I}_b]$ (right panel) obtained over $R=10^6$ independent replications, three different distribution of the underlying variables (Standard Normal, Uniform in $[0, 1]$ and Laplace centered in 0 with tail coefficient $a=1$), and sample size ranging from $N=100$ to $N=1000$. The confidence intervals in the plots are derived from the Chi-Square quantile function with $R-1$ degrees of freedom, $Q_{\chi^2}(q, R-1)$, under the assumption that the indexes are normally distributed. The bands represent a 95% confidence level and are obtained multiplying the computed value times $(R-1)Q_{\chi^2}(0.025, R-1)$ or times $(R-1)Q_{\chi^2}(0.975, R-1)$, respectively for the upper and lower bound.⁹

The results reported in the plots confirm that the behavior does not depend on the underlying distribution. While a clearly visible downward slope signals the presence of sub-asymptotic corrections, their effect is so small that they are already negligible, for any practical purpose, when the size of the sample is as small as $N=100$. This is generally below the size of the samples we consider to build CQTPMs in this paper. In the inferential analysis of the following sections, whenever an expression for the variance of the indexes is required, we use $C_s(10)=0.1120$ and $C_b(10)=0.0563$, corresponding to the average of the two coefficients over the three considered distribution for $N=1000$ (taken to the last significant digit).

5 Distributional analysis of firm growth persistence

We exploit the CQTPM framework developed above to condition out the possible dependence of growth rates on time-, country- and sector-specific effects, and firm size.

We first consider the observations on firm growth rates separately for each transition between year t and $t+1$ included in the sample time period, splitting them further by country and, within each country, by (2-digit)

⁹ In the case of $R=10^6$ replications, the bounds are 0.9972 and 1.0028.

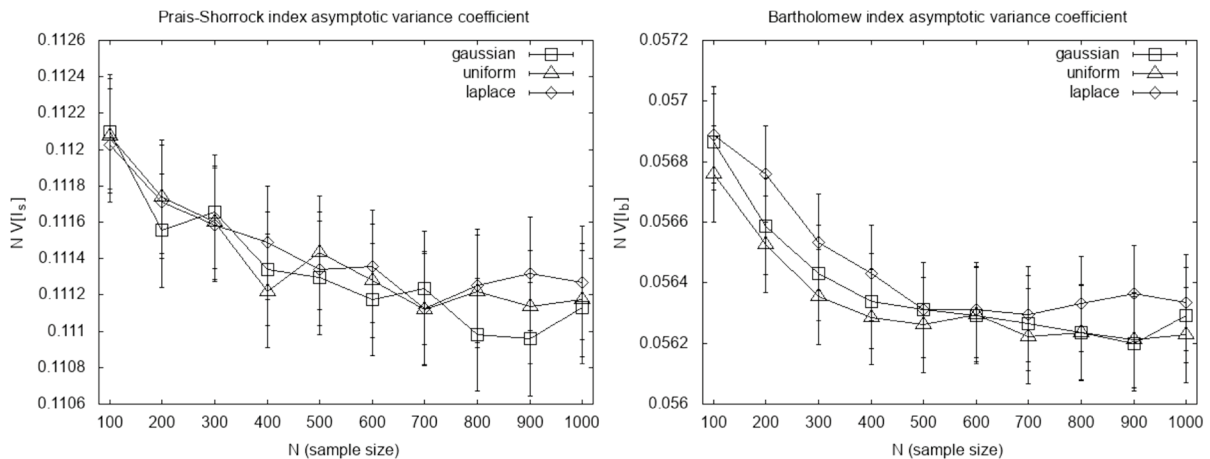


Fig. 2 Monte Carlo analysis of rescaled variance ($N V_N[\hat{I}]$, from Eq. 10), for the Shorrocks (left) and the Bartholomew (right) index. Figures obtained using $R=10^6$ replications of the respective index, each replication computed over a sample of

industries. These steps implicitly controls for spurious persistence in growth rates due to country, sector or time factors, due to the simple fact that these potentially omitted sources of dependence are naturally coded as discrete variables (they are discrete z variables, in the wording of Section 4.1). Then, within each of the transition-country-sector sub-samples thus obtained, we control for growth-size dependencies as follows. We take the triplet $\{g_{n,t}, g_{n,t+1}, s_{n,t}\}$, where $s_{n,t}$ is initial firm size (in terms of sales, in line with the definition of g), and build 5 equipopulated sub-samples according to the quintiles of the distribution of initial firm sizes (i.e., $L=5$ in Eq. 6 above). Within each size-quintile sub-sample, we then build a CQTPM by taking the deciles of the marginal empirical distribution of growth rates in t and $t+1$ as the initial and final states of the transition (i.e., $Q=10$). Lastly, we aggregate back the five QTPMs computed over the size-quintiles by averaging them, thus obtaining a CQTPM for each of the transition-country-sector sub-samples. In total, we are left with 600 CQTPMs to work with (5 countries, 20 2-digit sectors, 6 transitions).¹⁰

¹⁰ We tested the robustness of our results by experimenting with two different definitions of the CQTPMs. First, we conditioned upon initial size deciles instead of initial size quintiles. Second, we kept the conditioning on initial size quintiles as in the baseline analysis, but considered 20 equipopulated growth bins instead of growth deciles (re-computing the Monte Carlo values of C_c and C_b for $Q=20$). These alternative specifications did not significantly affect our main findings.

N independently drawn couples and using QTPMs with $Q=10$. For the generation of the underlying data we test three different distributions: Standard Normal, Uniform and Laplace

Conditioning on firm size is particularly important. As discussed in Section 3.2, growth rates depend on firm size in our data as they typically do in most previous studies. Simple, unconditional QTPMs computed ignoring growth-size dependencies, might show more persistence in some cells than in others just because similar sized firms are likely to exhibit similar growth rates in both the initial and final periods. In fact, Daunfeldt and Halvarsson (2015) and Capasso et al. (2013) report apparent differences between the QTPMs computed separately across firms belonging to different size classes. Instead of using ex ante defined size classes, as these previous studies do, our method of building CQTPMs adapts to the specific characteristics and evolution of the size distribution within each sector-country-transition sample.

5.1 Country-level analysis

We start by examining properties of CQTPMs at the country level. These are obtained by averaging the 20 sectoral CQTPMs obtained for each country in a given transition (controlling for firm size as described above), leaving us with 6 separate matrices to study for each country.

Figure 3 shows the CQTPMs corresponding to the 2016/2017 transition. Rows represent shares of firms that remain in the same decile or move to a different decile of yearly growth rates over the 2 years. Given the size of

the samples, the statistical estimation error of matrix elements is of the order of one thousand. We have thus only included significant digits in all our figures, while the gray scale helps in identifying the main patterns.

Two main results emerge, which are common to all countries. Firstly, there is a tendency to remain in the same decile, as shown by the darker stripe along the main diagonal. This tendency is stronger in the top and bottom deciles, revealing the presence of persistent over- and under-performing firms. Secondly, there is a relatively high probability of moving to the opposite decile, as suggested by the darker stripe along the secondary diagonal. In particular, there is a relatively higher probability of switching from Q1 to Q10, or vice-versa. This is evidence of sizable, anti-persistent bouncing effects affecting extreme growth events. Consistent patterns are also replicated in all the other transition years, in all countries (see online material, Appendix 2). The behavior observed in the extreme quantiles is in accordance with previous studies which examined unconditional TPMs. Our analysis confirms that they also survive after controlling for firm size and country, sector and time effects.

All these particular properties of growth dynamics would be totally invisible within the standard AR regression approach. Moreover, they actually show that the linear AR model discussed in Section 3.1 is a poor description of the growth dynamics. In fact, were the AR model able to provide a satisfactory characterization of the underlying process, a similar behavior would be seen across all initial states, that is across all the 10 rows of a matrix. This is clearly not the case in any of the countries considered.

Notice also that — here as in the rest of the paper — we do not present a pooled analysis aggregating CQTPMs over time. This would entail to average the CQTPMs obtained across the different transitions, thus implicitly assuming homogeneity of the matrices. However, we verified that the CQTPMs computed for the different transitions do not pass an homogeneity test (see Appendix 3 in the online materials). This may be a further source of bias in previous studies that apply TPMs or QTPMs to firm growth persistence, which often report matrices obtained after pooling data over time, without previously checking for homogeneity.

Although the qualitative analysis of the CQTPMs reveals interesting results, a central question remains: how do the intra-distributional movements observed

in the empirical CQTPMs compare with the benchmark of independent growth rates? Or, more technically, do the numbers observed in the CQTPMs cells represent a statistically significant deviation from the benchmark value of 0.1 that is expected under the null of independence? The theoretical and Monte Carlo results developed above about the properties of the indexes, allow us to address this key issue.

Recall that, since the various CQTPMs involve a different number of firms, the values of the indexes obtained from the observed CQTPMs cannot be compared across the different matrices. However, we can exploit that we have derived expected values and variance under the null of independence, for $Q=10$. We can thus compute the following standardized version of the indexes, as a function of the sample size

$$\tilde{I}_s = \frac{\hat{I}_s - E[\hat{I}_s]}{\sqrt{V_N[\hat{I}_s]}} = 3 \sqrt{N}(\hat{I}_s - 1), \tilde{I}_b = \frac{\hat{I}_b - E[\hat{I}_b]}{\sqrt{V_N[\hat{I}_b]}} = 4.22 \sqrt{N}(\hat{I}_b - 11/30). \quad (11)$$

These quantities are asymptotically distributed as a $\mathcal{N}(0, 1)$ and, thus, they are comparable across different samples and different matrices. By definition, their values indicate how many standard deviations the empirical data are away from their expected value under the null of independence. Negative values indicate that there is more persistence in the observed CQTPMs than under the null, while the opposite holds for positive values.

Table 4 reports un-standardized mobility indexes and their standardized version, computed for the country-level CQTPMs discussed above, by country and transition. The previous studies that apply mobility indexes to firm growth, only provided qualitative comparisons between unstandardized values and some theoretically meaningful benchmark (e.g., “full mobility”, corresponding to no firms remain in the initial quantile). The figures in Table 4 show that such comparisons might be deceiving, while the standardization procedure is essential in drawing definite conclusions about the nature of the process. In fact, consider the results for the Shorrocks index. If all firms remained in the same quantile over time (“no mobility”, i.e., main diagonal elements all equal to 1), we would expect to observe un-standardized values close to 0. Conversely, if no firms preserved their initial quantile (“full mobility”, i.e., all zeroes in the main diagonal), the expected value of the un-standardized index would be $10/9 = 1.1$. The

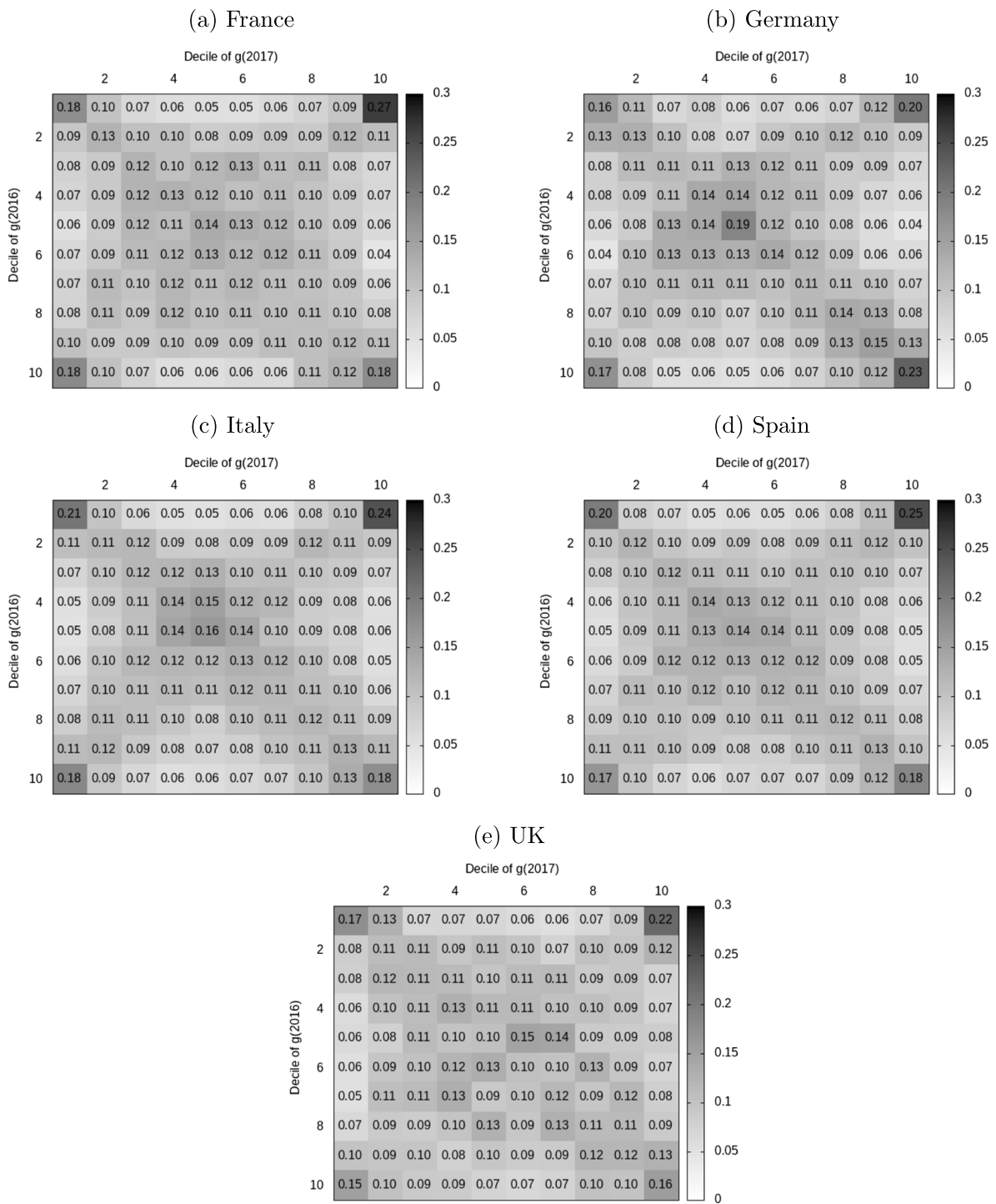


Fig. 3 Country-level Conditional Quantile TPMs defined on growth rates deciles, 2016/2017 transition

Table 4 Mobility indexes and test of the null of independence

| | Transition | France | Germany | Italy | Spain | UK | |
|---|-------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|----------------------|
| Shorrocks and Bartholomew mobility indexes (standardized values in parenthesis) computed on CQTPMs, by country and transition years. Asterisks refer to an F-test of the null hypothesis that the CQTPMs exhibit independence, implemented as an assessment of the distance between observed standardized indexes and their expected value under the null (1 for Shorrocks; ^{0.36} for Bartholomew). Significance level: ***($p < 0.01$) | Shorrocks | [2011/2012] | 0.967*** (-21.876) | 0.955*** (-14.382) | 0.959*** (-34.195) | 0.960*** (-28.237) | 0.985*** (-4.453) |
| | | [2012/2013] | 0.970*** (-19.809) | 0.934*** (-23.955) | 0.962*** (-34.441) | 0.964*** (-25.388) | 0.968*** (-9.244) |
| | [2013/2014] | 0.970*** (-18.486) | 0.934*** (-30.512) | 0.960*** (-34.513) | 0.964*** (-25.939) | 0.960*** (-11.659) | |
| | [2014/2015] | 0.969*** (-17.389) | 0.941*** (-30.312) | 0.958*** (-37.836) | 0.956*** (-31.715) | 0.969*** (-9.297) | |
| | [2015/2016] | 0.970*** (-15.145) | 0.936*** (-31.845) | 0.953*** (-44.355) | 0.958*** (-30.365) | 0.973*** (-7.897) | |
| | [2016/2017] | 0.966*** (-15.769) | 0.945*** (-25.576) | 0.956*** (-43.359) | 0.957*** (-30.762) | 0.974*** (-7.680) | |
| Bartholomew | [2011/2012] | 0.352*** (-13.462) | 0.339*** (-12.649) | 0.342*** (-28.561) | 0.338*** (-29.113) | 0.364 (-1.043) | |
| | [2012/2013] | 0.357*** (-8.769) | 0.335*** (-16.054) | 0.346*** (-24.635) | 0.347*** (-20.003) | 0.367 (0.286) | |
| | [2013/2014] | 0.358*** (-7.581) | 0.331*** (-22.805) | 0.345*** (-27.217) | 0.352*** (-15.056) | 0.332*** (-14.491) | |
| | [2014/2015] | 0.352*** (-11.497) | 0.336*** (-22.394) | 0.341*** (-31.943) | 0.344*** (-22.880) | 0.354*** (-5.420) | |
| | [2015/2016] | 0.354*** (-9.260) | 0.333*** (-23.659) | 0.345*** (-29.145) | 0.344*** (-23.097) | 0.359*** (-3.350) | |
| | [2016/2017] | 0.352*** (-9.177) | 0.332*** (-22.534) | 0.345*** (-29.687) | 0.350*** (-16.809) | 0.349*** (-7.135) | |

un-standardized values observed in our sample are not that far from this latter case, ranging between 0.94 and 0.98, while they are clearly far from zero. However, they are also not too far from the benchmark value of 1 that would emerge under the null of independence (recall Eq. 9). The standardized values allow to decisively discriminate between the two alternatives. In fact, they show that the un-standardized figures are several standard deviations smaller than under the null, revealing that there is more persistence in the data than an independent growth process would imply. Consistently negative standardized values in all countries and transitions suggest that this is a general pattern. To corroborate this, Table 4 also reports an F-test for the statistical significance of the difference between the observed standardized values and the value theoretically expected under the null of independence. All the F-tests document that the observed negative deviations from the benchmark are statistically significant.

The analysis of the Bartholomew indexes yields similar patterns and support consistent conclusions. The observed un-standardized Bartholomew indexes are all well above the value of zero expected under no mobility. They are also well below the value of $7/9 = 0.7$ which would be expected under max mobility, i.e., if all firms made the longest possible jump compared to their initial quantile.¹¹ In fact, the un-standardized values are very close to the value expected under the null of independence for $Q=10$, which is ^{0.36} (see Eq. 9). However, the standardized values and the associated F-tests reveal negative and statistically significant deviations from the null. This confirms that firm growth is more persistent than an

¹¹ Note that the theoretical maximum value of the Bartholomew index is a function of the number of quantiles Q . It equals $(3Q - 2)/(4(Q - 1))$ if Q is even, and equals $(3Q + 1)/(4Q)$ if Q is odd.

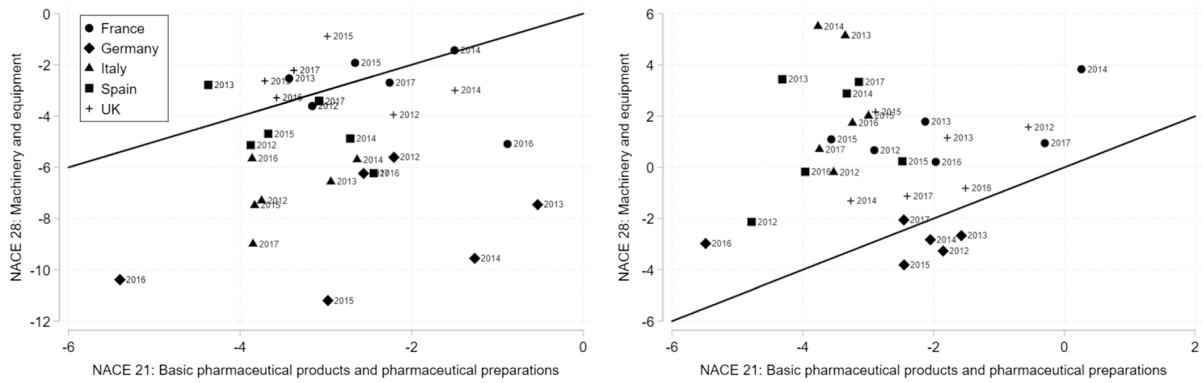


Fig. 4 Comparison of Standardized Shorrocks (left) and Standardized Bartholomew (right) in two selected sectors. Points refer to different countries and transitions. The line is the 45° sloping bisector

independent process, in all countries considered and across all transitions.¹²

Within this general result, the Shorrocks and the Bartholomew indexes also reveal differences across countries. The UK and, to a lesser extent, France have the least persistent growth rates, as the values of the indexes are less negative than in other countries. This ranking in the degree of persistence across countries, is essentially the same over all years in our data, although there is some variability within each country over time.

A notable difference in the findings between the two indexes is that the standardized Bartholomew indexes are generally less negative compared to the corresponding standardized Shorrocks indexes. That is, despite the general rejection of the null of independence, the Bartholomew indexes suggest more mobility (lower persistence) than the Shorrocks indexes do. This is compatible with the Bartholomew statistic giving more weight to off-diagonal movements and, thus, being more suited to account for the bouncing effects we observed in the top and bottom deciles of the CQTPMs.

Overall, the analysis of country-level CQTPMs supports the idea that firm growth intra-distributional dynamics are more persistent than an independent process would imply, even after controlling for biases that could arise due to dependence of growth rates on firm size and country-, sector- or time-specific factors.

¹² We checked that the main conclusions remain the same when the CQTPMs are computed over 3-year growth rates. Details and results are shown in Appendix 4 of the online material.

This emerges out of relatively low persistence in most CQTPMs cells, coupled with the specific dynamics in the corners of the matrices, characterizing firms that experience extreme relative growth episodes.

5.2 Sector-level analysis

The CQTPM framework we developed can also be exploited to investigate similarities and differences in firm growth persistence across sectors. As an example of the variety of sectoral patterns across the 600 2-digit sectoral CQTPMs we can work with, the scatter plots in Fig. 4 correlate the standardized Shorrocks and the standardized Bartholomew indexes computed by transition and country for two sectors, “Basic pharmaceutical and pharmaceutical preparations” (NACE 21) and “Machinery and equipment” (NACE 28).

Irrespective of the index considered, the plots generally reveal a good deal of sectoral heterogeneity. The range of values spanned by both indexes is wide and the points are scattered away from the 45° degree line, implying that the two sectors show different persistence levels. There are also apparent differences across the two indexes, likely due to the ability of the Bartholomew index to account for off-diagonal movements. According to the Shorrocks indexes, although both sectors display higher persistence than under the null of independence (negative standardized values), NACE 28 shows more persistent growth dynamics (less negative standardized value). The Bartholomew indexes, instead, feature various points where negative deviations from the null observed for NACE 21

are associated with positive deviations for NACE 28, in the same transition and country. This supports the hypothesis that firms in NACE 21 display more persistent dynamics than firms in NACE 28. Similarly heterogeneous patterns also emerge across other 2-digit sectors covered in our data. This evidence confirms our choice to compute CQTPMs at sector-level first, and not directly by country. In fact, it implies that sector-specific factors are an important source of variation in firm growth persistence, which one should condition out to avoid spurious estimates of transition matrices.

A further pattern emerging from Fig. 4 is that the points referring to the same country tend to cluster relatively close to each other. This suggests that country-specific heterogeneity, already observed in country-level CQTPMs, also features in individual sectors. Conversely, there is no significant clustering by transition year, suggesting that the variation due to time effects is relatively modest, once the sector and the country dimensions have been fixed.

To disentangle statistically the relative explanatory power of country, sector and time factors, we estimate the following variance decomposition regression model

$$\tilde{I}_{j,c,t} = \alpha + \delta_j + \delta_c + \delta_t + \epsilon_{j,c,t} \quad (12)$$

where the dependent \tilde{I} is either the standardized Shorrocks or the standardized Bartholomew index associated with the CQTPM of sector j in country c over the transition between $t-1$ and t , while the δ covariates represent the full sets of sector, country and transition fixed-effects.

Estimation results are reported in Table 5 for various specifications. When we include country dummies only (in model 1 and model 5, France is the baseline), the relative ordering of the coefficients is broadly in line with the results in Table 4 above. Italian and Spanish firms display more persistence than firms in other countries, while France and particularly the UK have a lower persistence. In Table 4, the sector-year CQTPMs of a given country were averaged and then the indexes computed. In contrast, in the regression model in Eq. 12, the coefficients associated to the country dummies are proportional to the averages of the sectoral indexes across all sectors within a given country. All values need to be interpreted as a

deviation with respect to the constant: the net value is negative for all countries in both indexes.

The estimates of models where we only include sector fixed-effects (model 2 and model 6, NACE 32 is the baseline) reveal that the coefficients on the sectoral dummies are more heterogeneous than the country dummy coefficients. They span a support of around 10, while the support of country dummies is about 5.6. In the regression on the Shorrocks indexes, considering the negative constant, all sectoral dummies have a negative net value. This confirms that there is generally more persistence than under the null of independence, with firms in NACE 10 (“Manufacture of food products”) having particularly persistent growth rates. On the other hand, in the regression taking the Bartholomew index as the dependent variable, NACE 28 (“Machinery and equipment”) is the only sector with positive net contribution.

The dummies relative to the transition years (see model 3 and model 5) are not significant, in line with the intuition from Fig. 4 that time provides a negligible contribution to the total variation.¹³

Taken together, the three sets of dummies explain 63.5% of the total variation in the Shorrocks indexes and 62.1% of the variation in the Bartholomew indexes (see models 4 and 8). Because of the symmetrical nature of the sample (same number of observations for each sector and same sectors for each country), the contribution of each group of dummy variables to the models’ R^2 can be computed by just dropping the other dummies. For both indexes, country and sector dummies together basically account for the entire explained variance. Country dummies account for 27.5% of the variance of the Shorrocks indexes and for 12.6% of the variance of the Bartholomew index. Sector dummies capture more: 35% of the variance of the Shorrocks indexes and 48.8% of the variance of the Bartholomew indexes.

Overall, the analysis of sector-level CQTPMs highlights that sectoral specificities are the main source of deviation of the observed persistence levels from the null of independent firm growth rates.

¹³ This finding suggests that growth dynamics are consistent over time, despite we reject homogeneity across QTPMs as shown in Appendix 3 in the online material.

Table 5 Decomposition of sector, country and time contribution to firm growth persistence

| | | Standardized Shorrocks | | | | Standardized Bartholomew | | | |
|---------|---------|------------------------|---------------------|--------------------|---------------------|--------------------------|---------------------|--------------------|---------------------|
| | | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
| Country | Germany | -1.478** (0.465) | | | -1.478** (0.344) | -1.448** (0.520) | | | -1.448** (0.338) |
| | Italy | -3.757** (0.515) | | | -3.757** (0.324) | -2.927** (0.610) | | | -2.927** (0.373) |
| | Spain | -1.915** (0.441) | | | -1.915** (0.238) | -1.678** (0.602) | | | -1.678** (0.424) |
| | UK | 1.908** (0.405) | | | 1.908** (0.309) | 1.216* (0.530) | | | 1.216*** (0.412) |
| Sector | 10 | | -6.223** (1.416) | | -6.223** (1.070) | | -8.995** (1.439) | | -8.995** (1.285) |
| | 11 | | 2.821** (0.526) | | 2.821** (0.498) | | 2.742** (0.699) | | 2.742** (0.574) |
| | 13 | | 2.367** (0.687) | | 2.367** (0.500) | | 1.801* (0.722) | | 1.801** (0.490) |
| | 14 | | 1.202 (0.933) | | 1.202 (0.642) | | 0.059 (0.771) | | 0.059 (0.577) |
| | 16 | | 2.124** (0.664) | | 2.124** (0.493) | | 3.529** (0.612) | | 3.529** (0.528) |
| | 17 | | 2.666** (0.635) | | 2.666** (0.464) | | 1.424* (0.622) | | 1.424** (0.440) |
| | 18 | | 0.247 (0.831) | | 0.247 (0.453) | | 0.273 (0.615) | | 0.273 (0.443) |
| | 20 | | 1.361* (0.683) | | 1.361** (0.486) | | -0.057 (0.684) | | -0.057 (0.510) |
| | 21 | | 3.284** (0.536) | | 3.284** (0.536) | | 2.136** (0.548) | | 2.136** (0.432) |
| | 22 | | 1.555* (0.629) | | 1.555** (0.387) | | 0.625 (0.678) | | 0.625 (0.471) |
| | 23 | | 1.846* (0.765) | | 1.846** (0.472) | | 3.222** (0.705) | | 3.222** (0.514) |
| | 24 | | 3.442** (0.710) | | 3.442** (0.517) | | 2.940** (0.580) | | 2.940** (0.458) |
| | 25 | | -1.183 (1.178) | | -1.183 (0.632) | | 3.411** (0.865) | | 3.411** (0.597) |
| | 26 | | 1.713* (0.736) | | 1.713** (0.593) | | 2.207** (0.583) | | 2.207** (0.465) |
| | 27 | | 2.094** (0.703) | | 2.094** (0.523) | | 2.404** (0.516) | | 2.404** (0.438) |
| | 28 | | 1.086 (0.741) | | 1.086* (0.525) | | 5.330** (0.616) | | 5.330** (0.659) |
| 29 | | 2.965** (0.615) | | 2.965** (0.477) | | 2.105** (0.554) | | 2.105** (0.417) | |
| 30 | | 3.931** (0.606) | | 3.931** (0.463) | | 3.558** (0.452) | | 3.558** (0.433) | |

Table 5 (continued)

| | | Standardized Shorrocks | | | | Standardized Bartholomew | | | |
|------------|-------------------|------------------------|---------------------|---------------------|---------------------|--------------------------|---------------------|---------------------|---------------------|
| | | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
| | 31 | | 2.695** (0.669) | | 2.695** (0.481) | | 3.712** (0.628) | | 3.712** (0.544) |
| Transition | [2011/2012] | | | 0.736 (0.477) | 0.736* (0.290) | | | 0.050 (0.538) | 0.050 (0.368) |
| | [2012/2013] | | | 0.351 (0.500) | 0.351 (0.332) | | | 0.612 (0.569) | 0.612 (0.412) |
| | [2013/2014] | | | -0.078 (0.525) | -0.078 (0.327) | | | -0.144 (0.525) | -0.144 (0.391) |
| | [2014/2015] | | | -0.248 (0.547) | -0.248 (0.318) | | | -0.403 (0.598) | -0.403 (0.381) |
| | [2015/2016] | | | -0.405 (0.551) | -0.405 (0.388) | | | -0.074 (0.559) | -0.074 (0.369) |
| | Cons | -3.700** (0.357) | -6.248** (0.512) | -4.808** (0.377) | -5.259** (0.458) | -2.264** (0.477) | -4.852** (0.450) | -3.238** (0.405) | -3.892** (0.441) |
| | Obs | 600 | 600 | 600 | 600 | 600 | 600 | 600 | 600 |
| | (R ₂) | 0.275 | 0.350 | 0.011 | 0.636 | 0.126 | 0.488 | 0.006 | 0.620 |

OLS estimates of Eq. 12. Baseline categories are: NACE 32 (Other manufacturing) for sector dummies; transition 2016/2017 for the transition dummies; France for country dummies. Bootstrap standard errors in parenthesis. Asterisks denote significance levels: *($p < 0.05$), **($p < 0.01$)

6 Sectoral determinants of growth persistence

The primary role of sector specific factors revealed by the analysis of sectoral CQTPMs, suggests firm growth persistence is strongly related to the characteristics of sectors. In this section we explore this relation further.

We correlate sectoral persistence, as measured by standardized indexes, to a set of sectoral variables which represent key features of structure and dynamics of industries, and are plausibly linked to patterns of firm growth. Specifically, we consider profitability, productivity, market concentration, business dynamism and openness to international markets. Profitability is defined as gross operating margins over total sales. Since we study manufacturing sectors, it makes more sense to focus on profits from operating performance, excluding the effect of financial assets and liabilities. Data are taken from the Structural Business Statistics (SBS) database maintained by EUROSTAT. We define productivity as labor productivity (LP), measured in terms of real value added per hour worked, available at the sectoral level from the EU-KLEMS database. As a measure of concentration, we

take the Hirschman-Herfindahl index, computed on sales of the firms operating in the same 2-digit sector in ORBIS (by year and country). Business dynamism is proxied via the churning rate, defined as the sum of firm birth and death rates per year in each sector, taken from EUROSTAT-SBS. Finally, we define openness as the ratio between the number of exporting firms and the total number of firms in a sector. Figures to compute this ratio are taken from EUROSTAT-Trade by Enterprise Characteristics (TEC) and EUROSTAT-SBS for the numerator and the denominator, respectively. Table 6 summarizes the definition of variables, their sources and coverage. Coverage varies according to whether it was possible to find complete information for all the 600 sector-country-transition combinations for which we computed the CQTPMs. The only problematic variable is openness, for which we have only 280 observations, due to the limited number of sectors reported in the TEC database.

The signs to be expected in the relationships between firm growth persistence and the sectoral characteristics considered are not all completely clear a priori, in particular for profits, productivity

Table 6 Sectoral characteristics

| Variable | Source | Description | #Obs |
|-------------------------|----------------------|---|------|
| Profitability | EUROSTAT-SBS | Gross operating surplus over sales turnover (%) | 597 |
| Labor Productivity (LP) | EU KLEMS | (log of) Value added per hour worked by persons engaged | 600 |
| Concentration | ORBIS | Herfindahl-Hirschman Index x 100 | 600 |
| Business dynamism | EUROSTAT-SBS | Sum of birth and death rates of firms (%) | 585 |
| Openness | EUROSTAT-SBS and TEC | Exporting firms over total firms (%) | 280 |

Sectoral variables are defined at 2-digit NACE level and by country, over the period 2011–2017

and openness. If high profitability levels in a sector are interpreted as a signal of market power, then high barriers to entry or the incumbents' ability to hamper competition should stabilize relative firm growth rankings and keep persistence higher, compared to low profitability sectors. On the other hand, high profitability in a given sector may indicate attractive investment/profit opportunities, which may be accompanied by substantial entry attempts and, as a consequence, increased turbulence and reduced persistence in growth rates. The relation with productivity is equally difficult to predict. In fact, high productivity sectors are environments where competitiveness is on average high, but this may lead to opposite predictions. On the one hand, since performing better than the average is difficult in such environments, one could expect relative growth and market shares to be more stable compared to low-productivity sectors. On the other hand, firms in highly competitive environments are arguably subject to stronger selective pressures, which is likely to increase turbulence. Concerning openness, one has to consider that involvement in international markets is both an opportunity and a threat to firms. Accessing export markets may help to sustain sales growth, especially when the domestic market is stagnant, and induce more stability in growth rates. At the same time, however, firms operating in more open sectors are also increasingly subject to adverse external shocks, and typically face fiercer competition. This may create turbulence in growth dynamics, resulting in a nuanced relation between openness and firm growth persistence. Conversely, sharper predictions seem possible about the role of concentration and business dynamism. We expect persistence in growth rates to be relatively higher in more concentrated sectors, since concentration is a signal of market power, either due to the structural characteristics of the sector or to

the anti-competitive behavior of incumbents, which counters changes in relative market shares over time. Lastly, persistence is naturally expected to decrease with business dynamism, since the sectoral turbulence due to entry and exit, by definition, involves instability in relative market shares.

To investigate empirically how sectoral characteristics relate with growth rates persistence, we estimate a series of regression models where the sectoral variables are regressed on the standardized mobility indexes associated with sectoral CQTPMs. To alleviate simultaneity, we take lagged sectoral variables. Specifically, the standardized indexes computed over the transition between $t-1$ and t , are regressed against sectoral variables measured in $t-2$

$$\tilde{I}_{c,j,t} = \alpha + \beta \mathbf{X}_{c,j,t-2} + \epsilon_{c,j,t}, \quad (13)$$

where \mathbf{X} is the vector of sectoral characteristics. Recall that negative standardized indexes indicate more persistence than under the null of independent growth rates. Thus, negative estimates of the β coefficients imply a positive association between sectoral characteristics and persistence.

Preliminary estimates where each variable is included alone in the model (not reported for space consideration), reveal that profitability, productivity and business dynamism show a statistically significant and negative association (positive β) with persistence, while openness does not display a statistically significant association. These findings emerge irrespectively of whether we consider the Shorrocks or the Bartholomew index as the dependent variable. Concentration also negatively associates with persistence, but the correlation is statistically significant only against the Shorrocks indexes.

In Table 7 we examine multivariate specifications, including all the sectoral variables together. The

Table 7 Firm growth persistence and sectoral characteristics

| | All variables | | | Excl. Openness | | |
|-------------------------------------|----------------------|---------------------|----------------------|---------------------|----------------------|----------------------|
| | (1) | (2) | (3) | (4) | (5) | (6) |
| Dependent: Standardized Shorrocks | | | | | | |
| Profitability | -0.023 (0.040) | -0.045 (0.038) | -0.038 (0.061) | 0.052 (0.023) | -0.082 (0.032) | -0.141* (0.036) |
| LP | 0.258* (0.085) | 0.560 (0.443) | 1.996** (0.602) | 0.019 (0.066) | 1.854** (0.327) | 1.415** (0.382) |
| Concentration | 0.046 (0.029) | 0.015 (0.017) | 0.012 (0.023) | 0.036 (0.021) | 0.022 (0.011) | -0.005 (0.012) |
| Business dynamism | 0.318** (0.055) | 0.144* (0.063) | 0.093 (0.079) | 0.277** (0.036) | 0.097 (0.039) | 0.078 (0.061) |
| Openness | 0.055** (0.015) | 0.064** (0.016) | 0.065** (0.025) | | | |
| Cons | -11.044** (1.240) | -8.291** (2.054) | -13.886** (3.104) | -8.990** (0.539) | -11.850** (1.651) | -10.060** (1.893) |
| Country FE | no | yes | yes | no | yes | yes |
| Time FE | no | yes | yes | no | yes | yes |
| Sector FE | no | no | yes | no | no | yes |
| R ² | 0.176 | 0.436 | 0.688 | 0.125 | 0.324 | 0.653 |
| Obs | 275 | 275 | 275 | 582 | 582 | 582 |
| Dependent: Standardized Bartholomew | | | | | | |
| Profitability | 0.004 (0.037) | 0.023 (0.055) | 0.010 (0.070) | 0.066 (0.027) | -0.018 (0.032) | -0.073 (0.042) |
| LP | 0.267* (0.087) | -0.378 (0.451) | 2.423** (0.653) | 0.062 (0.060) | 0.796 (0.361) | 2.173** (0.450) |
| Concentration | 0.019 (0.029) | 0.001 (0.024) | 0.018 (0.019) | 0.014 (0.017) | 0.003 (0.013) | 0.012 (0.012) |
| Business dynamism | 0.147 (0.065) | 0.042 (0.086) | 0.207 (0.097) | 0.150** (0.038) | -0.005 (0.062) | 0.167* (0.054) |
| Openness | 0.035 (0.022) | 0.049* (0.020) | 0.059 (0.031) | | | |
| Cons | -6.873** (1.659) | -2.170 (2.683) | -16.215** (3.557) | -5.975** (0.623) | -5.152 (2.301) | -13.510** (2.170) |
| Country FE | no | yes | yes | no | yes | yes |
| Time FE | no | yes | yes | no | yes | yes |
| Sector FE | no | no | yes | no | no | yes |
| R ² | 0.049 | 0.138 | 0.662 | 0.039 | 0.137 | 0.645 |
| Obs | 275 | 275 | 275 | 582 | 582 | 582 |

OLS estimates. All models include country, sector and transition fixed-effects. Bootstrap standard errors in parenthesis. Asterisks denote significance levels: * $p < 0.05$, ** $p < 0.01$

estimates in column 1 report baseline results without country, sector and time dummies. Results using the Shorrocks index as the dependent variable, reveal that persistence decreases (positive coefficients) with

productivity, business dynamism and openness. The regression with the Bartholomew index confirms a significant and inverse relation (positive β) between persistence and productivity.

We then extend the model by adding fixed-effects, to control for the various sources of unobserved heterogeneity in the data. Given our interest in sectoral characteristics, inclusion of sector fixed-effects needs to be carefully considered. Indeed, sector fixed-effects may absorb much of the statistical significance of the relations, if sectoral characteristics vary mostly across sectors, rather than within sector. Column 2 only includes country and time fixed-effects. Business dynamism and openness display a statistically significant coefficient vis-a-vis the Shorrocks indexes, while openness is the only variable showing a statistically significant association with the Bartholomew indexes. The positive coefficients of these variables confirm that they are associated with reduced persistence. Since identification works across sectors, the findings highlight the association between standardized indexes and sectoral variables *in deviation* from their average values computed across industries, within country and transition year. Consequently, the positive coefficient on business dynamism, for instance, means that the sectors where churning is above the average sectoral churning observed in a country in a given transition, display lower persistence than the average sectoral persistence observed in a country in a given transition.

In estimates in column 3 we also add sector fixed-effects. The identification of parameters exploits the deviation of indexes and regressors from their within-sector specific average, computed within country and transition year. In the specification with the standardized Shorrocks indexes, productivity and openness are the only statistically significant variables. They are both associated with reduced persistence. The same holds true for productivity, but not for openness, when considering the regression on the Bartholomew indexes.

In columns 4–6, we perform a robustness check excluding openness from the models. As mentioned, there are no data on openness for about one half of the 600 sector-country-year combinations for which we can compute CQTPMs and associated mobility indexes. The estimates on the other sectoral characteristics might be biased, if the sector-country-time combinations where we can observe openness, are systematically different. The estimation results show that this is not generally the case. We broadly confirm the conclusion from the baseline estimates that productivity and business dynamism display statistically significant association with persistence, while concentration does not.

Overall, productivity, business dynamism and openness to trade stand out as the variables that have the most stable patterns of statistical significance. They all tend to display an inverse relation with firm growth persistence.

7 Discussion and conclusions

We have applied CQTPMs and related mobility indexes to draw precise inference on persistence in intra-distributional dynamics of firm growth rates, exploring its determinants across sectors and countries. The analysis is based on a sample of manufacturing firms active in four major European economies and the UK over the period 2010–2017.

Our first and main finding is that, although there is more persistence than under an independent growth process, a good deal of intra-distributional mobility characterizes firm growth dynamics. This result contributes to the long-standing debate about the validity of Gibrat's classical model of firm growth and the "illusion of randomness" (Henderson et al., 2012; Derbyshire and Garnsey, 2014; Coad et al., 2015). In fact, strictly speaking, our analysis supports a rejection of any model of firm growth based on independent growth shocks. At the same time, however, our evidence conflicts with theories predicting high stability in growth rates rankings, induced by fitter firms experiencing sequences of positive growth events and less fit firms continuously shrinking over time. In this respect, our results resonate with previous evidence that growth rates are, if not totally erratic, at least quite difficult to predict (even with machine learning algorithms, see Coad and Srhoj (2020)).

As a qualification of the general finding of considerable turbulence in intra-distributional dynamics, CQTPMs display more persistence for relatively fast-shrinking and fast-growing firms. Persistence in the top of the growth rates distribution is supportive of the attention that high-growth firms receive in the literature. However, it is precisely in the extreme deciles that we also find evidence of anti-persistent, bouncing effects, entailing that firms in the extreme deciles are likely to experience large jumps to the opposite extreme deciles of the distribution. These episodes of extreme volatility and reversal document a good deal of lumpiness in growth processes, whereby large (positive or negative) adjustments tend to be followed

by periods of relative inaction. Previous studies on firm growth TPMs relate this particular dynamics in extreme quantiles to the relative abundance, in those quantiles, of small and hence more volatile firms. Our analysis of CQTPMs show that such dynamics represent a pervasive property of the growth process, which is still present even after conditioning on firm size (and also on country, time and sector). These patterns also relate to the stylized fact that growth rates distributions exhibit thick tails. Frequently occurring large growth shocks (positive or negative) are not just due to the presence of a fixed set of top and bottom performing firms. They also result from significant intra-distributional mobility, volatility and bouncing effects.

From a policy perspective, the considerable turbulence in firm growth patterns that we document, is hardly good news for policies targeting the growth of specific groups of firms and aimed to achieve long-lasting effects. As there are generally low chances that firms growing in a given period will steadily grow over time, growth policies are likely to have a volatile and transient effect. In particular, the bouncing effects observed in the top deciles, lend additional support to previous studies showing that high-growth firms are often “one-hit wonders”. Conversely, instability of growth rates rankings over time is good news for anti-trust policies. In the sectors and countries analyzed, there seems to be no serious concern for a tendency toward strong cumulative growth, potentially leading to excess dominant position in markets. At the same time, however, the growth process does not appear to naturally contribute to a gradual reduction of market concentration.

Our findings also highlight the role of sectoral factors as drivers of firm growth persistence. Previous studies have provided qualitative evidence on variation of TPMs properties at the aggregate country level or at the level of specific sectors, typically without conditioning on size. Our multi-country, multi-sector analysis of CQTPMs, simultaneously conditioning on firm size and time-, country- and sector-specific factors, reveals that sectoral specificities explain considerably more variation than country-specific and time-specific factors do. That is, growth rate persistence does not primarily depend on country context and institutions, nor does it vary significantly with specific year-by-year contingencies. Rather, it correlates significantly with some structural characteristics of sectors, such as productivity, business dynamism, and openness to trade. This provides an initial basis to inform about how industry performance and dynamics

may interplay with policies targeting firm growth persistence, in case the latter is seen as a target for policy.

There are several extensions of the analysis which we did not consider in this study, mostly due to the characteristics of the available data, yet which seem particularly promising for future research.

Firstly, our methodology could be extended to consider further potentially confounding factors that may create spurious persistence in intra-distributional dynamics. Conditioning on the sector, country, and time effects, and on firm size — as we have done here — seemed a natural starting point. Growth-size dependencies play a prominent role in firm growth theory and empirics, while sector, country and time effects are obvious sources of dependence in growth patterns. Future research could include additional conditioning. The analysis in Coad et al. (2018), although the authors do not examine CQTPMs, suggests that firm age could be a particularly interesting candidate. But any other firm-level characteristics potentially related to firm growth — such as efficiency, innovativeness, or access to finance — could be easily integrated into the procedure to obtain conditional matrices. Also, with larger data (e.g., covering the entire population of firms in a country) one could take a more fine-grained disaggregation of sectors to compute the CQTPMs. This would enable to focus on the intra-distributional dynamics occurring across firms that can be more reliably assumed to compete in the same market, compared to the 2-digit sectors we have used here. The CQTPMs framework we developed is flexible enough to be adapted to all these cases. The only caveat is that more and more data points are required whenever an additional conditioning is applied.

Another interesting avenue for further research relates to the time scale over which intra-distributional dynamics are studied. Given the relatively short sample period available, we mainly focused on yearly growth rates and 1-year transitions and just examined a robustness check confirming our main results for 3-year growth rates. However, with longer-in-time data, our methodology could be exploited to explore persistence over longer transitions (e.g., 5-year or 10-year growth) and/or over longer time horizons (e.g., over several decades). Theories of firm-industry dynamics that reject independence, predict stronger persistence in relative growth over the short-run, due to capabilities and firm-specific attributes being relatively slow-changing variables. Yet, those models abstract from a clear definition of the time scale over which their predictions unfold. For instance, do firms’ structural

advantages apply over yearly growth rates or over longer time scales, such as 3–5 years of growth? How long is the short-run? 5 years or perhaps a decade? By gathering empirical evidence on growth persistence over different time scales, one could verify over which time scales the theoretical predictions are more consistent with the data. That would help better judge those theories. It would also help to connect the empirics of firm growth persistence with genuinely long-run theories, which predict successive phases of stability and instability in industry dynamics, according to Schumpeterian creative destruction vs. creative accumulation, and over the development of the product life-cycle.

With its limitations and opportunities for further development, this work provides a benchmark framework for future research on firm growth persistence. It highlights the importance of conditional analysis and it shows how to address it, allowing to assess how much persistence remains in the data, once firm growth dynamics is cleaned from the underlying confounding factors. Also, the proposed framework makes a crucial step forward, from a purely qualitative and possibly misleading analysis of TPMs toward formal inference. We applied the new framework to analyze persistence in relative market success as proxied by sales growth, but the same analysis can be easily adapted to study persistence using other proxies of firm size, such as employment or fixed assets. More generally, the proposed methodology could apply in any area of research in firm dynamics, not confined to firm growth studies, as a tool to investigate the extent of intra-distributional persistence in key dimensions of firm performance.

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