Performance Indices and Tuning in Process Control

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Abstract—**Industrial practice in process control shows a very wide gap with respect to the theory, as usually taught at the university. Standard controller tuning is often a compromise between performance and robustness. In this paper we examine some common PI(D) controller tuning rules and the achievable performances for a PI(D) controlled plant modeled as a first order plus time delay. The performance indices considered are IAE and ISE in a particular non-dimensional form including also controller output variations. The output performance indices are weakly dependent on loop gain.Therefore loop gain is essentially constrained by robustness, while the indices are dominated by the frequency response. Conversely the controller output variations are strongly dependent on the loop gain.**

I. INTRODUCTION

IT is commonly claimed that there exists a wide gap between theory and practice in process control. In this paper we will address this problem in attempting to narrow the gap. In particular we will clarify the use of performance indices and tuning of standard regulators as a compromise between performance and robustness.

PID control has remained the most popular one in the industry since the 1950s and has formed a separate and important research topic, which is difficult to address in the general control framework for its output feedback nature [1] and has caused remarkable activities recently [2]–[3].

Among many new published books on PID, O'Dwyer in [4] (see the book review [5]) collects 245 tuning rules (104 for PI and 141 for PID), organizes them in an unified and ordered form and reports two Tables illustrating the publishing activity on tuning rules and PID control since 1942.

Despite the importance of process control, many industrial control loops operate poorly. In testing thousands of control loops in hundreds of operating plants, Techmation Inc. and others have found that more than 30% of the control loops actually increase variability over manual control due to k/k_c [6]. In addition, pulp and paper mill auditing has shown that a large percentage of control loops actually de-stabilize product uniformity as a result of either valve stiction or poor controller tuning [7].

Van Overschee and De Moor [8] report that 30% of PID controllers were in manual, and for those in auto mode, 25% had the default parameters (never been tuned!), 80% are badly tuned, and 30% increased short-term variability.

One reason why so many control loops operate poorly is that there are often more than 1000 loops in large process plants and not enough control engineers to maintain every loop [6]. Another reason may be the gap between theory and practice. Indeed with so many tuning rules, what rule is the best rule?

Many tuning rules have been proposed for standard controllers and processes modeled as first order plus time delay transfer function (FOPTD), but is this model adequate?

It may happens that a tuning rule works well for the model but it does not work for the real plant, as well a simulation of a controller with the FOPTD model is unsatisfactory, but the same controller gives good results, if applied to control the real plant.

Therefore, there appears to be a strong need for evaluation of the closed loop performance. Assessment and monitoring of controller performance has been an active area of research for the last decade [9]–[10].

A characterization often used in practice to assess control system performance is the integral of the absolute value of the error (IAE):

$$IAE = \int_0^\infty |e(t)| \cdot dt \qquad (1)$$

where the error is the set-point minus the output.

Shinskey [11] argues that the IAE value is a good economic performance measure, because the size and length of the error in either direction is proportional to lost revenue.

On the other hand this performance index is analytically intractable, so that theoreticians prefer more tamed indices such as ISE (integral of the squared error):

$$ISE = \int_0^\infty e(t)^2 \cdot dt \qquad (2)$$

or IT^{2}SE [12]:

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$$IT^2SE = \int_0^{\infty} t^2 e(t)^2 \cdot dt \tag{3}$$

In this paper we restrict our attention to performance indices IAE and ISE and to the tuning rules proposed by Lopez [13], Rovira [14], reported in well-known textbooks on Process Control [15] – [16], along with the tuning rules proposed by Zhuang-Atherton [12].

II. PRELIMINARY RESULTS

The plant is assumed in the standard form of a FOPTD (first order plus time delay) transfer function, typical of process control:

$$P(s) = \frac{A \cdot e^{-s\theta}}{1 + sT} \tag{4}$$

and the standard controller is:

$$C(s) = k_p \left[1 + \frac{1}{sT_i} + \frac{sT_d}{1 + s\frac{T_d}{N}} \right] \tag{5}$$

where $T_d = 0$ for a PI controller, $N = \infty$ for a ideal PID, $N \geq 10$ for a realizable PID (N=10 is the usual choice).

The open loop transfer function is G(s) = P(s)C(s) . For a given P(s) and a particular choice of C(s) we have a critical gain k_c and a critical frequency f_c, so that

$$\omega_c = 2\pi f_c, \qquad k_c G(j\omega_c) = -1 \tag{6}$$

Harriott in [17] suggested to follow the rule $\omega_c IAE =$ constant; on the other hand Jarek (as quoted in [18]) noted that with an optimal choice of the controller:

$$(ISE)_{optimal} = 4.1 \cdot \omega_c^{-1} k_c^{-1.6} \quad \text{for a PI controller} \tag{7}$$

$$(ISE)_{optimal} = 3.5 \cdot \omega_c^{-1} k_c^{-1.9} \quad \text{for a PID controller} \tag{8}$$

We tried to compute (by using a Montecarlo approach) a formula for the performance indices, with the following structure:

$$ISE \text{ or } IAE = c_0 \cdot \omega_c^{c_1} k_c^{c_2} \tag{9}$$

Simulation parameters were randomly chosen in the following intervals: $1 \leq A \leq 10$, $1 \leq T \leq 10$, $\theta = 1$, (such that $0.1 \leq \theta / T \leq 1$).

The results of simulations with controller based on the classic tuning rules due to Zhuang-Atherton [12], Lopez [13] and Rovira [14] are shown in the following Table I.

TABLE I

Evaluated IAE-based index	Rule
$IAE \cdot \omega_c = 7.8 \cdot k_c^{0.5}$	Zhuang-Atherton (ideal PID)
$IAE \cdot \omega_c = 1.73 \cdot k_c^{0.6}$	Zhuang-Atherton (real PID)
$IAE \cdot \omega_c = 4 \cdot k_c^{1.2}$	Lopez (ideal PID)
$IAE \cdot \omega_c = 2.8 \cdot k_c^{0.15}$	Lopez (real PID)
$IAE \cdot \omega_c = 3.56 \cdot k_c^{0.3}$	Rovira (ideal PID)
$IAE \cdot \omega_c = 1.9 \cdot k_c^{0.6}$	Rovira (real PID)

Note that in [12] the optimal tuning rules are reported only for the ISE performance index; in this case all results in the Tables referred to IAE are computed by using the controller tuning rules suggested as optimal rules for ISE.

TABLE II

Evaluated ISE-based index	Rule
$ISE \cdot \omega_c = 3.6$	Zhuang-Atherton (ideal PID)
$ISE \cdot \omega_c = 1.68$	Zhuang-Atherton (real PID)
$ISE \cdot \omega_c = 2.4 \cdot k_c^{-0.4}$	Lopez (ideal PID)
$ISE \cdot \omega_c = 2.0 \cdot k_c^{-0.2}$	Lopez (real PID)
$ISE \cdot \omega_c = 2.38 \cdot k_c^{0.5}$	Rovira (ideal PID)
$IAE \cdot \omega_c = 2.17 \cdot k_c^{-0.3}$	Rovira (real PID)

These formulas show a large variability of the coefficient c_2 appearing as exponent of k_c. This fact may be confusing for process engineers and poses challenging questions for theoreticians; indeed, if all results claimed to describe the optimal solution, why the solution is not unique?

In the following section we will have a closer look at the dependence of the performance indices on the loop gain.

III. PERFORMANCE AND LOOP GAIN

As in the previous section, we tried to obtain the relationship between performance indices and the normalized loop gain k/k_c in cases of the tuning rules of Zhuang-Atherton, Lopez and Rovira.

By defining the following indices:

$$IAE \cdot \omega_c = J_1, \qquad ISE \cdot \omega_c = J_2 \tag{10}$$

we obtained the results shown in Figs.1–13, reporting J_1 and J_2 as functions of k/k_c. Each plot includes 10 curves for 10 θ / T ratios (from $\theta / T = 0.1$ up to $\theta / T = 1$, step 0.1.

We note that all curves have a similar aspect, and may be described as:

$$J_{1,2} = a\frac{k_c}{k} + b\frac{k_c}{k_c - k}. \tag{11}$$

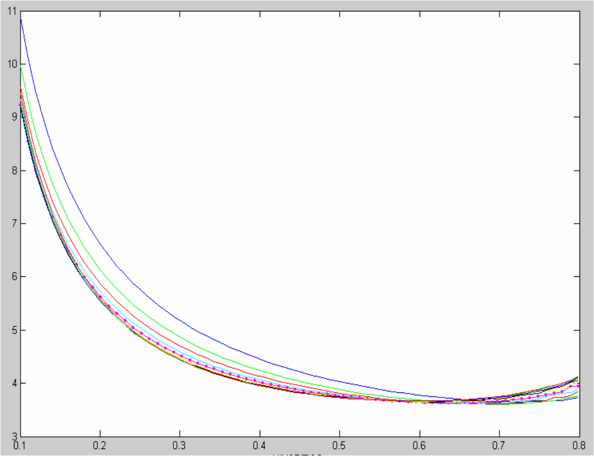

Fig. 1. J_2 as a function of k/k_c. Ideal PID controller tuned according to Zhuang-Atherton.

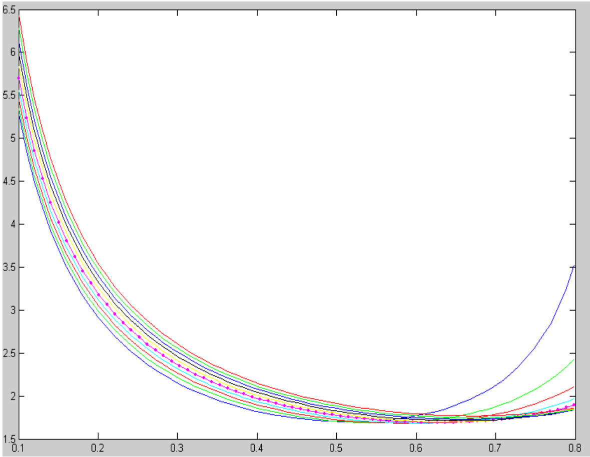

Fig. 2. J_2 as a function of k/k_c. Realizable PID controller tuned according to Zhuang-Atherton.

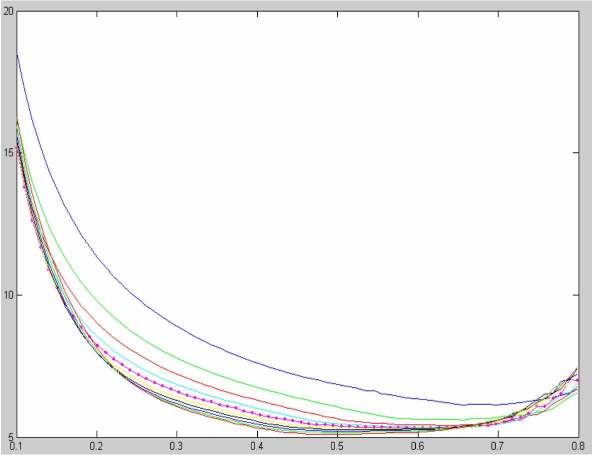

Fig. 3. J_1 as a function of k/k_c. Tuning rules of Zhuang-Atherton for the ideal PID controller.

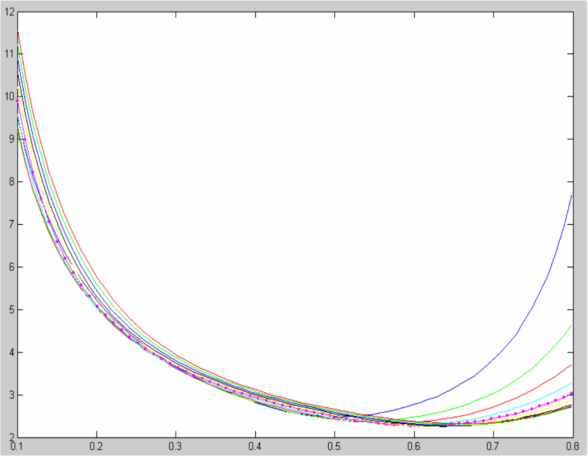

Fig. 4. J_1 as a function of k/k_c. Realizable PID controller tuned according to Zhuang-Atherton.

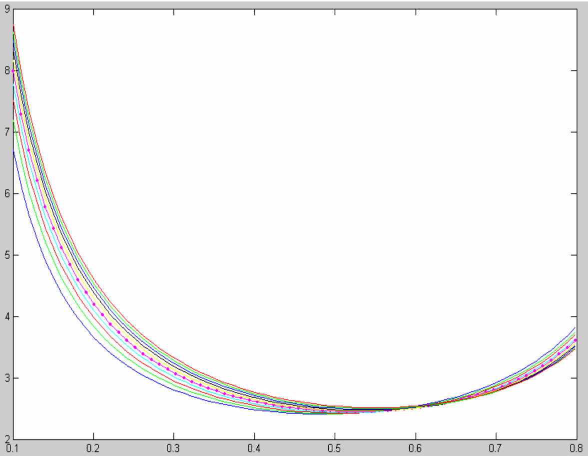

Fig. 5. J_2 versus k/k_c. PI tuned according to Zhuang-Atherton.

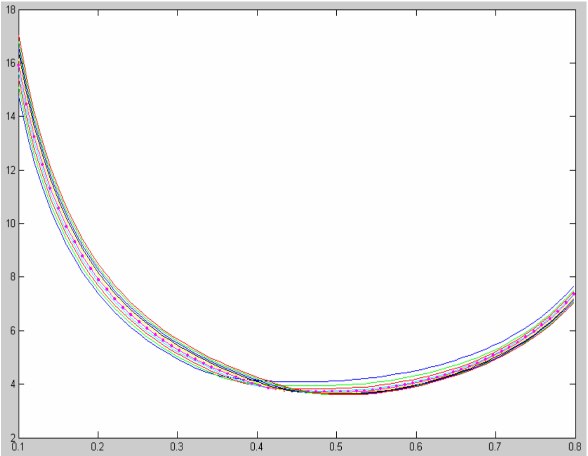

Fig. 6. J_1 versus k/k_c. PI tuned according to Zhuang-Atherton.

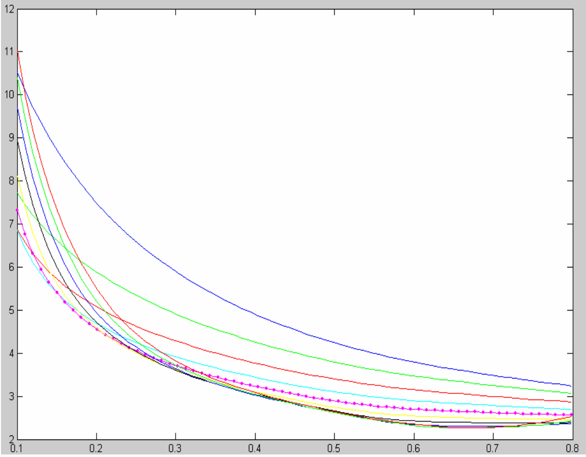

Fig. 8. J_1 as a function of k/k_c. Realizable PID tuned according to Lopez.

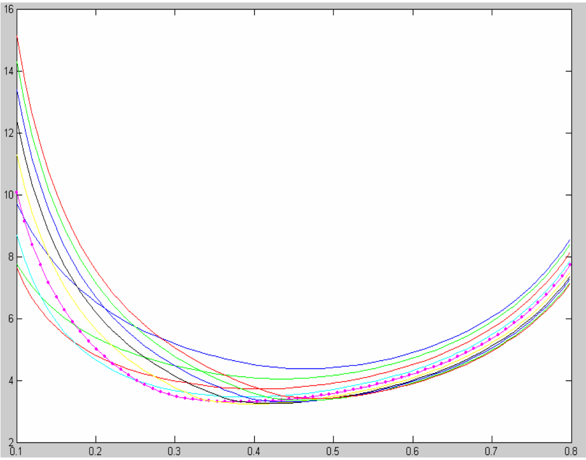

Fig. 9. J_1 versus k/k_c. PI tuned according to Lopez.

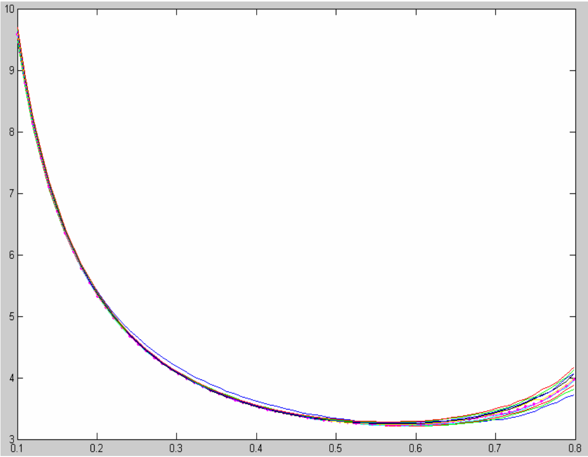

Fig. 10. J_2 as a function of k/k_c. Ideal PID tuned according to Rovira

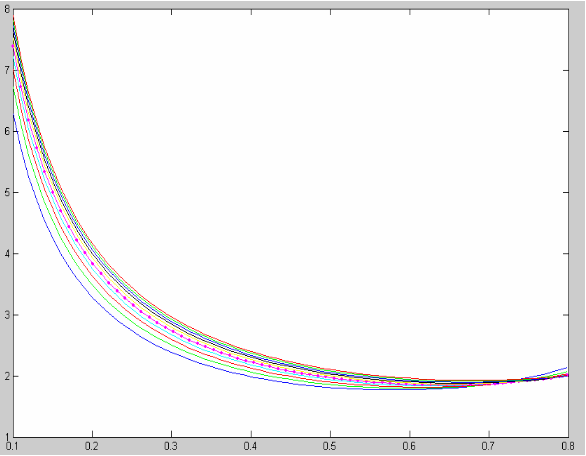

Fig. 11. J_2 as a function of k/k_c. Realizable PID tuned according to Rovira.

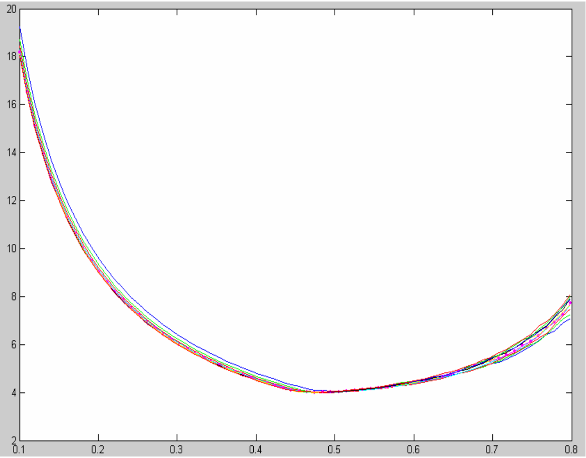

Fig. 12. J_1 versus k/k_c. Ideal PID tuned according to Rovira

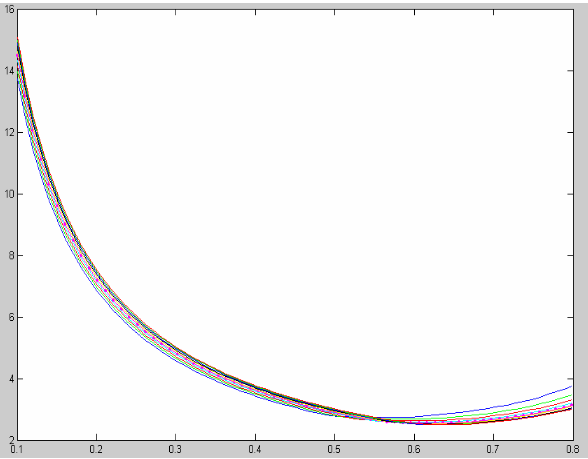

Fig. 13. J_1 versus k/k_c. Realizable PID tuned according to Rovira

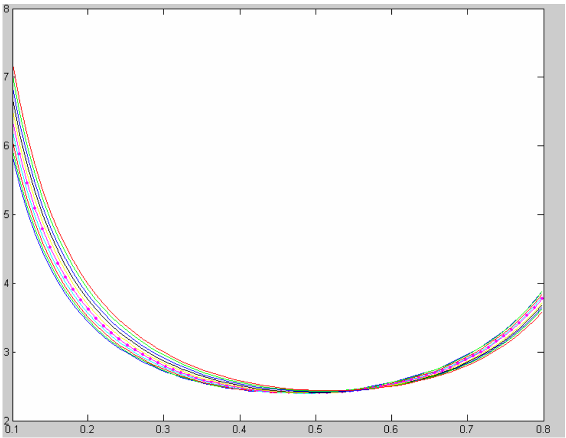

Fig. 14. J_2 versus k/k_c. PI tuned according to Rovira

TABLE III

Coefficient a	Coefficient b	θ / T
1.3362	0.55617	0.1
1.1655	0.61995	0.3
1.095	**0.66272**	**0.5**
1.0577	0.69288	0.7
1.0473	0.70807	0.9

In Table III coefficients a and b are shown for different θ / T ratios in case of a mean square interpolation of the curves shown in Fig. 1. In bold a practical suggestion for choosing the interpolating coefficients of (11) is put into evidence. Similar tables can be extracted from each figure.

The optimal values of k/k_c and J can be computed as:

$$\frac{k^*}{k_c} = \frac{1}{1+\sqrt{b/a}} \tag{12}$$

$$J_{1,2}^* = \left(a+\sqrt{a/b}\right)\cdot\left(1+\sqrt{b/a}\right) \tag{13}$$

TABLE IV

$a \div b$	IAE TUNING RULES
$1.5695 \div 1.0032$	Zhuang-Atherton (ideal PID)
$0.9583 \div 0.3118$	Zhuang-Atherton (real PID)
$1.4190 \div 1.2908$	Lopez (ideal PID)
$1.3423 \div 0.20036$	Lopez (real PID)
$1.3527 \div 0.94896$	Rovira (ideal PID)
$1.3423 \div 0.20036$	Rovira (real PID)

If we restrict our attention to the range $\theta/T > 0.2$, we note that all the curves are almost coincident, so that we can assume as representative the curve for $\theta/T = 0.5$. The coefficients a, b in Tables IV-VI are computed by assuming $\theta/T = 0.5$, $k/k_c=0.3$, and $k/k_c=0.7$. These representative values are reported in the Tables IV-VI. These remarks apply to all cases, with an error less than 8% in the range $0.3<k/k_c< 0.7$. The only exceptions in some cases are found applying the tuning rules of Lopez.

TABLE V

$a \div b$	ISE TUNING RULES
$1.09500 \div 0.66272$	Zhuang-Atherton (ideal PID)
$0.59897 \div 0.26632$	Zhuang-Atherton (real PID)
$1.00800 \div 1.04160$	Lopez (ideal PID)
$0.56562 \div 0.26321$	Lopez (real PID)
$0.96567 \div 0.62303$	Rovira (ideal PID)
$0.71188 \div 0.2588$	Rovira (real PID)

TABLE VI

$a \div b$	PI TUNING RULES
$1.12520 \div 0.9779$	Zhuang-Atherton (IAE)
$0.68714 \div 0.5472$	Zhuang-Atherton (ISE)
$0.44235 \div 1.4023$	Lopez (IAE)
$0.58631 \div 0.61198$	Lopez (ISE)
$0.96567 \div 0.62303$	Rovira (IAE)
$0.57316 \div 0.61198$	Rovira (ISE)

IV. CONTROLLER OUTPUT VARIATIONS

In the previous sections it appears that in the comparison of the three different tuning rules, based on IAE and ISE indexes, the Rovira tuning rules are particularly attractive. A question on the controller output variations must be approached: what is the quality of the control action in terms of actuator preservation? In a real plant the controlled variables must be limited both in its amplitude (saturation bounds) and in its dynamics (finite bandwidth of the actuators). In most PID control loops valves are the only components with moving parts and poor controller tuning has high costs in terms of valve wear and maintenance programs. In [19] a quantitative analysis of how tuning affects valve costs is described. Therefore an accurate study of the overall performance can be performed by considering also a performance index based on the derivative action of the controller output, the $IA\dot{U}$:

$$IA\dot{U} = \int_0^\infty \left|\dot{u}(t)\right| \cdot dt \tag{14}$$

It represents a measurement of the variability of the control action and hence a $IA\dot{U}$ minimization corresponds to the requirements of smooth and bandwidth limited actuators for reducing wearing and maintenance costs. A sligthly different index has been proposed in [20] in order to evaluate the aggressiveness of PI actions. Consider the application of (14) to a process in the form (4) and evaluate the results obtained by varying the θ/T ratio in the range 1:10). In the hypothesis of a controller based on Rovira rules and IAE performance index, measuring the critical gain k_c, the critical pulsation ω_c of the controlled system and the gain k of the system in the presence of different gains k_p of the proportional controller, we obtained the results shown in Fig.15. Plots are drawn as a function of the ratio k/k_c and the index J_u was considered, where

$$\frac{IA\dot{U}\cdot\omega_c}{k_p}=J_u \tag{15}$$

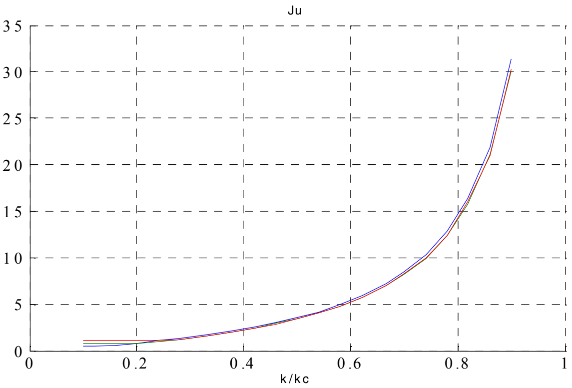

Fig. 15. J_u versus k/k_c. for 3 θ/T values (0.3, 0.5, 0.8) and PI tuned according to Rovira rules.

We note again that all curves have a similar aspect, and may be described as:

$$J_u=\frac{3.42\cdot\dfrac{k}{k_c}}{1-\dfrac{k}{k_c}} \tag{16}$$

Formula (16) is an optimal approximation of the plots drawn in Fig.14, in the range 0.2<k/k_c< 0.7. It may be noted that it is independent from the θ/T ratio and that the J_u values are low if k/k_c is limited.

V. SUGGESTIONS AND WARNINGS

In the previous sections we have examined the interplay between performance and robustness for some common PI(D) tuning rules applied to plant modeled as first order plus time delay systems. Of course, these rules may be a poor and small sample with respect to 245 rules in [4]; however our results validate a conjecture by [17] and may be representative of a quite general behaviour. Main results obtained in our investigation can be summarized as:

1) the normalized performance indices J are well represented by (11);
2) the normalized performance indices J are weakly dependent on k/k_c, if 0.3 < k/k_c < 0.7. As shown in Figs.1–14, the curves are so flat that it is misleading to get the optimal setting of k by a minimization of J analytically, or, so much the worse, in real time by an on-line procedure. In such misleading case, it might be possible to get a value of k near to k_c, without a significant increment of the performance index; therefore it is better to use poor or default tuning, than to choose the "optimal" gain.
3) If k/k_c is chosen for robustness, the performance index IAE or ISE effectively shows an inverse dependence on ω_c [16], where ω_c is the pulsation corresponding to a real negative value of the open (and closed) loop transfer function. Therefore the best performance is achievable above all by a suitable reshaping of the frequency response.
4) The performance indices have been renormalized by using the factor ω_c, instead of 1/θ [21]. This choice is more suitable in practice, because ω_c may be easily obtainable by a relay test [22], whereas θ, the apparent time delay, is only a trick adequate for modeling.
5) In control loops, valves are the only components with moving parts and poor controller tuning has high costs in terms of valve wear and maintenance programs. Therefore it is very important to evaluate the relative performance by using the index (15). The dependence of this index on the parameters has been evaluated as in (16).
6) The overall performance index may be chosen by the designer as a suitable combination of the indices (11) and (16). The relative weights are application dependent.

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