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ENDOGENOUS NETWORKS IN RANDOM POPULATION GAMES

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Population learning in dynamic economies traditionally has been studied in contexts where payoff landscapes are smooth. Here, dynamic population games take place over "rugged" landscapes, where agents are uncertain about payoffs from bilateral interactions. Notably, individual payoffs from playing a binary action against everyone else are uniformly distributed over [0, 1]. This random population game leads the population to adapt over time, with agents updating both actions and partners. Agents evaluate payoffs associated to networks thanks to simple statistics of the distributions of payoffs associated to all combinations of actions performed by agents out of the interaction set. Simulations show that: (1) allowing for endogenous networks implies higher average payoff compared to static networks; (2) the statistics used to evaluate payoffs affect convergence to steady-state; and (3) for statistics MIN or MAX, the likelihood of efficient population learning strongly depends on whether agents are change-averse or not in discriminating between options delivering the same expected payoff.

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1. INTRODUCTION

Interaction-based models were notably used to study decentralized economies composed of consumers and/or firms that repeatedly interact over time (Kirman, 1997; Fagiolo, 1998). Interactions come from externalities because the outcome of the decision of an agent depends upon the behaviors of other agents in the population (Blume and Durlauf, 2001). If individual expectations are modelled as myopic or adaptive, individual decisions depend on past observed behaviors of others actors. Therefore, any aggregate measure of the behavior of the system (e.g., some statistics computed on individual choices) follows a Markovian process (Brock and Durlauf, 2001).

The most investigated instances of interaction-based models are *dynamic population games*. A dynamic population game (Blume, 1993) is an interaction-based description of a decentralized economy where agents play non-cooperative, simple games against other agents in the population. Interactions and externalities are modelled through payoff matrices describing the outcome of bilateral games. Agents have myopic or adaptive expectations about the behaviors of their opponents in the games and use simple boundedly rational decision rules to choose their strategies. Examples range from deterministic and myopic best-response to stochastic rules such as the log-linear rule or the best-reply with noise. A key assumption is that behaviors are reversible. Therefore, agents can, from time to time, revise their current choices.

The study of dynamic population games raises three related questions: (1) Under which conditions does the system converge in the long-run? (2) Which efficiency properties do long-run aggregate outcomes possess?[1] (3) To what extent direct interactions, as well as individual rationality, affect population dynamics and long-run properties?

Such questions have been addressed in two different interaction settings. First, many scholars have studied population games where the *interaction structure*, defining who plays with whom at each time, does not evolve through time. The basic exercise is to assess how different interaction structures affect the long-run behavior of the system. Interaction structures range from *global* ones, where each agent has a positive

[1]For instance, in population games where all agents play a coordination game, one is often interested in studying whether the system converges to a configuration where all agents play the Pareto-efficient Nash equilibrium against a risk-dominant one (Kandori, Mailath, and Rob, 1993).

probability of playing the game with any other agent, to *local* ones, where each agent always plays the game with her "nearest neighbors." In local structures, players are placed in some metric space (e.g., regular lattices) and interact only with agents located in their neighborhoods. The spatial dimension of the economy reflects some underlying socio-economic dissimilarity, defined in a space of unobserved variables assumed to change very slowly compared to the pace at which individual actions are revised. The assumption of a static interaction structure is justified because the frequency at which agents update their partners in the game is so small that this additional revision process would not affect the properties of the one governing strategy updating[2].

Second, a complementary and more recent line of research has originated from the observation that, when agents change their partners at a frequency that is comparable to that at which they update their strategies, it becomes crucial to study the interplay between the two revision processes in a co-evolutionary manner (Goyal and Vega-Redondo, 2001; Jackson, 2003; Droste, Gilles, and Johnson, 2000; Fagiolo, 2004). Such models describe dynamic settings where agents have the option of repeatedly updating both the strategy and the set of their partners[3]. Network updating becomes endogenous and often occurs on the basis of expected payoffs in different alternative networks. Once again, a crucial assumption concerns whether agents can freely select any other agent in the population, in a sort of global matching process, or can only adapt to the set of their interacting partners locally, in some pre-defined neighborhood structure.

Dynamic population games have focussed upon the roles of agents' rationality and of the structure of interactions, starting from relatively simple strategic situations, such as pure coordination games, "prisoners' dilemma," and "hawk-dove." However, they have ignored economies where the payoff landscape generated by individual stage-games played by agents is not smooth. In all these settings, individual payoffs are common knowledge, there is no uncertainty about payoffs, and each agent plays the same game against any other agent in the population. Moreover, the learning process acts at a population-level on landscapes where the payoff of any single agent depends on some average levels of the behaviors of players belonging

[2]Kandori, Mailath, and Rob (1993), Young (1996) and Young (1998) study dynamic games where agents play against everyone else in the population, with interaction structures global and static. On the contrary, spatial games with locally interacting players and static networks are investigated in Ellison (1993), Young (1998), Nowak, Bonnhoefer, and May (1994) and Nowak, Bonnhoefer, and May (1994).

[3]See also Goyal and Janssen (1997), Skyrms and Pemantle (2000) and Mailath, Samuelson, and Shaked (2000). Dynamic models of non-cooperative network formation only, without simultaneous choice of a strategic variable, are studied in Bala and Goyal (2000), Watts (2001) and Jackson and Watts (2002).

to her interacting set. For instance, in population coordination games, individual payoffs are a linear function of the total number of coordinated agents in the network. This implies that individual payoffs are invariant to permutations preserving the frequency of agents currently playing a given strategy in the network. As a result, the payoff landscape is relatively smooth because it does not change very much across configurations characterized by the same distribution of local frequencies of agents playing coordinated actions across different networks.

On the contrary, in many real-world settings, agents facing much less smooth payoff landscapes, are uncertain about which game to play with whom in any time period and, consequently, about the payoff that they might expect from any bilateral interaction. If the type of each prospective opponent is unknown, at least at the beginning of the process, and if agents can change the game they play both over time and in different encounters, the metaphor of a smooth payoff landscape with homogenous stage-game payoff matrices can be misleading (Bednar and Page, 2002; Calvert and Johnson, 1997; Taylor, 1987). Such situations are characterized by a strong heterogeneity of stage-game payoffs matrices and, in turn, a high variability of payoffs experienced by agents after each bilateral game. Specifically, expected and actual payoffs of each economic agent can be sensitive to small changes in the configuration of actions currently performed by actors in her network.

We suggest a preliminary model to capture population game settings where agents face high uncertainty about expected payoffs from bilateral interactions. Further we suggest that, at least as a first approximation, individual payoffs from playing a certain strategy, given the current configuration of population choices, can be modelled as being i.i.d. random variables. We assume that if an agent interacts with everyone else in the population, the payoff she receives, conditional to any possible combination of actions performed by the others, is distributed as a uniform i.i.d. random variable with support [0, 1]. We call this setting a "random population game."

We do not model how agents learn "to play the right game," while they discover the "types" of their opponents (cf. Bednar and Page (2002) for an alternative approach based on individual learning). On the contrary, we study how the population adapts over time, when agents can adjust *both* their actions *and* their network. We assume that from time to time agents can either delete or add links[4]. Agents hold Markovian expectations based only on last-period observations and use deterministic best-reply rules so as to maximize their expected payoffs. Maintaining a link is assumed costless and link addition or deletion require mutual consent. Finally, we

[4]Network updating rules are similar to those in Jackson (2001), Watts (2001) and Jackson (2003). Unlike these models, however, we impose mutual consent in both link addition and deletion.

allow payoff tie-breaking rules to account either for *change-averse* players (who always stick to current choice when a tie occurs) or *change-lover* players (who always accept to change even if their payoff does not change).

In order to compute expected payoffs associated to local networks where a player *does not* interact with all others, we assume that players use simple statistics computed on the distribution of payoffs associated to all possible combinations of actions performed by agents outside their local networks. We will study four simple statistics: average of such payoffs (MEAN henceforth), their maximum value (MAX), their minimum value (MIN) and a random draw thereof (RND). These rules are heuristics used by individuals to form expectations on the payoff associated to new environments. A rule is understood as the way agents cope with the uncertainty of the system. For example, if an agent uses a MIN rule to compute the expected payoff of the network associated to a link deletion, she is "conservative" or "pessimistic."

The above criteria have a direct interpretation in terms of the trade-off between mean and spread of experienced payoffs. Since the cardinality of the set of payoffs associated to all possible choices of agents outside the network *increases* as the total number of her links *decreases*, the distribution of the expected payoffs of an agent using a MIN criterion becomes *concentrated* around a *decreasing mean* as the agent reduces her network. Conversely, the expected payoffs of an agent using a MAX criterion become *concentrated* around an *increasing mean* as she keeps deleting links. Therefore, different criteria will have non trivial consequences on individual and population-wide fitness landscapes.

We study the long-run behavior of the model in different setups. First, we explore the case where *networks are exogenously given and time-invariant*[5]. Our original contribution here is to compare systems characterized by an increasing average total number of links and different payoff criteria (MIN, MAX, MEAN, and RND). Simulations show that if the average number of links is sufficiently large, then no steady-state is ever reached. All populations further explore the landscape, and the serial correlation between average payoffs is not significant. The long-run relationship between the distribution of individual payoffs and the total number of links held by agents is driven, for any payoff rule, by the trade-off between average and spread. The set of payoffs associated to all possible choices of agents outside the network becomes larger because an agent reduces her total number of links. Then the variance of the payoffs decreases with the total number of links, for a mean of 0.5.

[5]In this case, the model has a structure similar to that of Kauffman's NK class of formalizations (Kauffman, 1993), but also with some substantial differences which we will discuss below.

Second, we explore random population games with endogenous networks. We run Montecarlo simulations to investigate the effect of initial network and strategy configurations, payoff criteria and tie-breaking rules on long-run average outcomes. We show that both the long-run convergence to steady-states and the short-run dynamic properties are affected by the payoff rule and whether players are change-averse or not. We find that if agents use the MEAN rule, then, irrespective of the degree of change-aversion, the system has many steady states. Populations climb local optima by first using action- and network-updating together and then network-updating only. Climbing occurs through successful adaptation and generates long-run positive correlation between the total number of links and average payoffs. With MIN or MAX rules, the long-run behavior of the system is instead affected if players are change-averse. If they use the MIN rule, then the network converges to a steady-state where all agents are almost fully connected but strategies are not, so that average payoffs oscillate. If agents use the MAX rule then the system has many steady-states, in both networks and actions, characterized by few links and different levels of average payoff. If agents are change-lovers, then the population can explore a larger portion of the payoff landscape. With agents using the MIN rule, the network converges to completion, but exploration on strategies goes on forever. Under the MAX rule, the system has a unique optimum. All populations converge to the same payoff distribution but neutral network updating will continue forever, without affecting realized payoffs.

In Section 2 we briefly discuss the relevant pieces of literature and introduce the model presented in Section 3. Simulation results are presented in Section 4. Finally, Section 5 concludes and summarizes the main results.

2. POPULATION GAMES, PAYOFFS AND ENDOGENOUS NETWORKS

Dynamic population games are games played over time by large populations of boundedly rational players. The standard framework common to dynamic population games consists of a set of N individuals who play a game in discrete time. At any time $t \geq 1$, each individual i plays a strategy $s_i^t \in S$. If $S = \{-1, +1\}$, two agents i and j play a bilateral game whose symmetric payoff matrix is given by:

$$G = \begin{bmatrix} a & b \\ c & d \end{bmatrix},$$

with $a, b, c, d \in R$.

All agents play the same game, they know that the others do the same and that G is common knowledge. Assume that at each time, any agent i plays a bilateral game defined by G against all agents j belonging to the set

$V_i^t \subseteq I$. The collection $(V_i^t)_{i \in I}$ is called the *interaction structure* at time t. At any t, an agent i is chosen[6] at random from I to revise her current state (s_i^t, or V_i^t, or both). Agent i then forms Markovian expectations about her next-period payoff under different actions or interaction structures, and takes a decision rule to choose her next-period state, either s_i^{t+1}, or V_i^{t+1}, or both. Decision rules differ in the introduction of idiosyncratic noise, interpreted as the possibility of experimentation or mistakes. In the absence of noise, the rule is deterministic and is usually based on local best-reply dynamics. For instance, if V_i^t are exogenously given and do not change over time, agents will pick their next-period strategy by taking the rule:

$$s_i^{t+1} = \underset{s \in \{+1,-1\}}{\arg\max}\, w(s_i s_j^t, j \in V_i^t)$$

where total individual payoffs $w(s_i s_j^t, j \in V_i^t)$ to i from interacting with agents in the network are defined as the sum (or the average) of all payoffs from bilateral games. A stochastic term can reverse, with some small probability, the decision maximizing local payoffs. The "mistake" can in turn be either constant as the relative frequencies of players choosing $+1$ or -1 in each V_i^t change (*noisy best-reply* rules, Ellison (1993)) or state-dependent (e.g., the *log-linear* rule, where the probability of choosing against the majority becomes very small, although not null, as the "majority" grows larger (Brock and Durlauf, 2001; Blume, 1993)).

A dynamic population game is therefore completely defined with the payoffs in the matrix G and the interaction structure at each time, or the rule that governs how the interaction structure changes over time.

Most[7] models are focussed on either *coordination* or *prisoners' dilemma* games with *static* interaction structures, where $V_i^t = V_i\ \forall t \geq 1$. However, a few contributions have pointed out that endogenous network dynamics might strongly affect long-run equilibrium patterns and their efficiency properties[8].

[6]This scheme is known as *asynchronous* updating. The consequences of assuming *synchronous* updating schemes, where agents perform a parallel updating, or *incentive-based* updating schemes, where agents revise their choices depending on the state of the system, are studied in Page (1997).

[7]Exceptions are provided by Nowak and May (1993), Nowak, Bonnhoefer, and May (1994) and Herz (1994).

[8]These contributions posit dynamic population games where agents, from time to time, have access to a network updating decision, either deterministic or noisy, and often modelled as a best-reply rule. A crucial ingredient is to know if agents choose their next-period network V_i^{t+1} in comparing expected payoffs from every possible network (Goyal and Vega-Redondo, 2001) or they just have the option of adding or deleting a small number of links in any choice-stage (Jackson, 2003). In this latter case, agents choose any other agent in the population as a new partner with a positive probability (Droste, Gilles, and Johnson, 2000) or must choose their networks with respect to some underlying geographical structure (Fagiolo, 2004).

In coordination games, "static interactions" yield a robust foundation for equilibrium selection. Whenever the underlying 2×2 game G has two Nash equilibria, one of which is Pareto efficient and the other is risk-dominant, Kandori, Mailath, and Rob (1993), Young (1996), Blume (1993), and Ellison (1993) showed that the unique long-run equilibrium is the risk-dominant one and that local interactions can speed up the rate of convergence. Convergence to an efficient outcome is the case when non-exclusive conventions are assumed, so that agents can pay to remain flexible, choosing not to choose (Goyal and Janssen, 1997); or when players are mobile (Bhaskar and Vega-Redondo, 1996; Ely, 1996; Oechssler, 1997; Dieckmann, 1999)[9]; or when interaction structures are endogenous and agents are free to select any partner in the population (Goyal and Vega-Redondo, 2001). On the contrary, when geographical barriers isolate individuals from each other, population learning is more likely to converge to risk-dominant equilibria (Fagiolo, 2004).

Similar results were obtained in dynamic population games where agents play prisoners' dilemmas[10]. No matter whether interactions are static or evolving endogenously, the iterated prisoner dilemma played by Markovian agents allows cooperation to be sustained, unlike in standard game-theoretic models. If players can refuse interactions with other individuals, and interaction structures become partly endogenous, the population tends to cooperate more than in the case of a compulsory prisoner dilemma (Zimmermann, Eguluz, and San Miguel, 2001; Hanaki and Peterhansl, 2002).

The analysis of decentralized economies where agents face more complicated payoff structures remains to be done. Indeed, the baseline model of a dynamic population game shares the assumptions that all agents know that everyone plays the same game and stage-game payoffs are common knowledge. This implies that the payoff landscape where the population learning takes place is quite smooth, so that individual payoffs vary little as the frequency of agents playing the same strategy slightly changes. Therefore, the long-run behavior of the system is directly interpretable in terms of bilateral games played by indi-

[9]The selection of partners is made endogenous by assuming the existence of a fixed number of spatial locations. Players are mobile and can select their future partners by choosing the place they want to move to, on the basis of the expected net payoff each location.

[10]Dynamic prisoner dilemma (DPD) population games with static interaction structures were studied in Axelrod (1984), Herz (1994), Nowak and May (1993), Nowak, Bonnhoefer, and May (1994), Oliphant (1994), Oltra and Schenk (1998) and Tieman, Houba, and van der Laan (1998). Population games where agents play DPD and can choose not to interact with an opponent (PD with ostracism or refusal) were instead investigated by Hirschleifer and Rasmusen (1989), Kitcher (1993), Smucker, Stanley, and Ashlock (1994), Stanley, Ashlock, and Tesfatsion (1994), Ashlock, Smucker, Stanley, and Tesfatsion (1996) and Hauk (1996).

viduals. For example, in coordination population games, the purported agent homogeneity of stage-games and the nature of the game to be played imply that the payoff to any agent i playing, say, $+1$ is a linear function of the total number of agents in i's network currently playing $+1$. Individual payoffs are invariant to any permutation of individual choices that keeps local frequency constant. Hence, the aggregate state of the system, when all play $+1$, is a state of maximum coordination.

2.1 The Model: An Informal Description

We account for situations in which agents may ignore the game their opponents will play in the next encounter, and in which stage-game payoffs can be uncertain or change endogenously. Thus, we model environments where network payoffs capture the interdependencies between agents and the high sensitivity of payoff landscapes to changes in the strategies of other players.

We consider a population of $N \geq 3$ agents, $I = \{1, \ldots, N\}$, and a binary strategy space $s_i^t \in \{-1, +1\}$. Agents live in a world where everyone is connected with anyone else. Their payoffs change in unpredictable ways whenever a change in the current configuration of the system occurs, that is whenever an agent changes either her strategy, or her partners, or both.

Suppose, for the sake of exposition, that $N=3$. Consider first the payoff landscape defined over the complete-network population game. Since the payoff of any agent i depends on the choices of the remaining $k_i - 2$ agents, each one faces $2^{N-k_i+1} = 4$ possible configurations for each $s_i^t \in \{-1, +1\}$ and thus $2 \cdot 2^{N-k_i+1} = 8$ individual payoffs $\pi_i(s_1^t, s_2^t, s_3^t)$. Uncertainty is rendered by assuming that $\pi_i(s_1^t, s_2^t, s_3^t)$ are i.i.d. and uniformly distributed over the interval [0,1][11]. Table 1 presents an example of a complete payoff landscape, i.e., random drawings of possible payoffs for all combinations of strategies in the case of a fully connected network. Actual payoffs depend on the links an agent maintains with the others.

Links are bilateral and costless. A player can hold any number of links between 0 (isolated agents) and $N - 1$ (complete connectivity), and the

[11]If the system adapts using an asynchronous updating mechanism and individual best-reply decision rules, population dynamics are similar to standard adaptation over rugged fitness landscapes in Kauffman's (1993) NK model where $K = N - 1$. Kauffman uses global payoff (fitness) to drive adaptation: a new configuration is chosen if its global fitness is higher. On the contrary, we use local payoff criteria: a link is established or deleted, and a strategy is switched, if the payoff of the single agent(s) involved (single bits) increases, regardless of the rest of the population (rest of the string). This difference has important consequences on the dynamics and in particular on the likelihood of lock-in into local optima. In Kauffman's model, the assumption of complete connectivity is taken to reflect the highest possible level of ruggedness of the fitness landscape where the population adapts.

TABLE 1 An Example of Payoff Landscape with $N=3$. (s_1^t, s_2^t, s_3^t) are the Combinations of Strategies that the Three Agents can Play; π_1, π_2 and π_3 are Individual Agents' Payoffs

(s_1^t, s_2^t, s_3^t)	π_1	π_2	π_3
$(-1, -1, -1)$	0.56	0.11	0.24
$(-1, -1, +1)$	0.77	0.54	0.17
$(-1, +1, -1)$	0.58	0.31	0.45
$(-1, +1, +1)$	0.55	0.59	0.19
$(+1, -1, -1)$	0.78	0.42	0.70
$(+1, -1, +1)$	0.32	0.54	0.25
$(+1, +1, -1)$	0.04	0.44	0.78
$(+1, +1, +1)$	0.80	0.67	0.44

total number of links can differ among agents, so that the resulting graph is not homogeneous. Given the payoff landscape $\pi_i(s_1^t, s_2^t, s_3^t)$, each agent is fully characterized at each time period by her current choice s_i^t and the set $V_i^t \subseteq I$ containing all her current partners.

If a link between i and j exists, experienced payoffs of both i and j are affected by the choice of the partner. Therefore, if agent 1 is connected with both 2 and 3 and plays, say, $s_1^t = -1$, she will face a payoff which changes as soon as either one of her partners switches her strategy, according to $\pi_i(s_1^t, s_2^t, s_3^t)$ entries in Table 1. If an agent, conversely, is not connected with all $N-1$ other agents in the population, she will use a simple statistics in order to compute expected payoffs from playing a given strategy. This statistics will be computed on payoffs associated to all combinations of strategies that could be played by agents outside the current network. For example, suppose that $s_1^t = -1$ and that the statistics employed by agent 1 is the arithmetic mean (MEAN)[12]. If 1 is currently connected with 2 only, then she faces only two possible payoffs, according to what agent 3 plays. Each payoff is computed as the MEAN of payoffs associated to any possible choice for s_3^t:

$$w_1(-1; s_2^t) = \begin{cases} \frac{1}{2}(0.56 + 0.77) & \text{if } {s_2}^t = -1 \\ \frac{1}{2}(0.58 + 0.55) & \text{if } {s_2}^t = +1 \end{cases}.$$

If 1 is currently connected with no other player, her payoff will be:

$$w_1(-1) = \frac{1}{4}(0.56 + 0.77 + 0.58 + 0.55)$$

Finally, if agent 1 considers payoffs associated to choosing $+1$ and connecting with 3 only, she gets:

[12]Alternative choices are MAX, MIN or RND. The latter consists in picking one of the 2^{N-k_i+1} payoff entries at random.

$$w_1(-1;s_3^t) = \begin{cases} \frac{1}{2}(0.78+0.04) & \text{if } {s_3}^t = -1 \\ \frac{1}{2}(0.32+0.80) & \text{if } {s_3}^t = +1 \end{cases}$$

Assume that at time $t=0$ a random initial configuration (s_i^0, V_i^0) for $i=1,\ldots,N$ is given. We study a population dynamics governed by an asynchronous updating process both on strategies and bilaterally on local networks. At each $t\geq 1$ a pair of agents h and k is randomly drawn to attempt graph updating given the current strategy configuration. If a link connecting h and k already exists, the link is tentatively removed, if it does not, the link is tentatively added. Decisions are made by best-responding deterministically to current local configurations under the different alternatives. We consider two different tie-break rules. In network updating *without neutrality*, a link is removed or added if and only if both agents are strictly better-off after the change. This implies that agents are change-averse, because they prefer to stick to their current choices unless the alternative option is associated to strictly larger payoffs for both. In network updating *with neutrality*, a change is accepted even if it implies the same payoff that both agents experienced before the change. This implies that agents are change-lovers, because they prefer to change their current choices even if the new one is associated to the same payoff for both[13].

Next, an agent, say h, is drawn at random to update her current strategy s_h^t given the network configuration obtained in the first stage. We assume that agent h, best replying deterministically to the current local configuration, chooses the strategy s that strictly maximizes $\pi_h(s; s_j^t, j\in V_h^t)$, while keeping s_h^t if a tie occurs.

Consider the case in which the current configuration is $(+1,\ +1,\ +1)$ and suppose that agent 1 is connected with 2 but not with 3, who is isolated. If 1 and 3 are drawn for a network update, they contemplate the possibility of forming a link between them. Agent 1 compares her current payoff $\frac{1}{2}(0.04+0.80)$ with the payoff after link addition, namely 0.80. Agent 3 compares her former payoff $\frac{1}{4}(0.17+0.19+0.25+0.44)$ with the possible new one $\frac{1}{2}(0.25+0.44)$. In this case both agents are strictly better off. under the change and the link is added. After network updating, player 3 is connected with 1 only. If she is called to strategy updating, she switches to -1 since $\frac{1}{2}(0.70+0.78) > \frac{1}{2}(0.25+0.44)$.

These updating rules define a population dynamics whose long-run outcome (absorbing state, cycle, etc.) and effciency properties (aggregate payoff, across-agents distribution, etc.) depend upon the criterion

[13]The term *neutrality* refers to the fact that, under the associated tie-breaking rule, agents accept *neutral* changes, they add or delete a link even if they both continue to get the same payoff. Decisions are made on the basis of payoffs observed by agents at the time of the choice.

employed to form expectations over networks (as well as by whether or not updating rules permit *neutrality*).

We now formally present the model and study it through simulations.

3. THE MODEL

Consider a fixed population of agents $I = \{1, 2, \ldots, N\}$, $N \geq 3$ and assume that time is discrete. At any time t, an agent $i \in I$ is characterized by her current strategy $s_i^t \in \{-1, +1\}$ and the set of agents of her opponents in the game $V_i^t \subseteq I - \{i\}$. The underlying links connecting agents are bilateral: $i \in V_j^t \Leftrightarrow j \in V_i^t$, and maintaining a link is costless for both agents. The size of V_i^t is $k_i^t = |V_i^t \in K = \{0, 1, 2, \ldots, N-1\}$ and let $\overline{V}_i^t = V_i^t \cup \{i\}$. The sets $\{V_i^t,\ i \in I\}$ induce a non-directed graph $G_t \in P(N)$, where *P(N)* is the set of all non-directed graphs over *I*. We denote $ij \in G^t$ the fact that in the network G^t agents i and j are linked. At any time, the system is characterized by the pair $\{\Omega^t, G^t\}$, where $\Omega^t = (s_i^t)_{i \in I} \in \{-1, +1\}^N$.

A random population game is defined through the following payoff structure. Let $u_i^t(\Omega^t, G^t)$ be the payoff to agent i at time t, given the current state of the system. For any subset $J \subseteq I$, define a J–restricted configuration of actions by $\Omega^t(J) = (a_j^t)_{j \in J}$. We assume that the payoff to i at t does not change if the strategies currently performed by all agents in I who are not partners of i under the current graph G_t change:

$$u_i^t(\Omega^t, G^t) = \pi_i^t(\Omega^t(\overline{V}_i^t)) = \pi_{i,t}(s_i^t; s_j^t, j \in V_i^t),$$

To model uncertain environments, we assume that if all agents were connected with anyone else, or $\overline{V}_i^t = I, \forall i \in I$; or, equivalently, if G^t were completely connected: $k_i^t = N - 1, \forall i$, then:

$$\pi_i^t(\Omega^t(\overline{V}_i^t)) \sim X,$$

where X are i.i.d. random variables with p.d.f. F. From now on, we assume that $X \sim U\,[0, 1]$.

To form expectations about payoffs $\pi_i^t(\Omega^t(\overline{V}_i^t))$ associated to local networks $\overline{V}_i^t$ such that $\overline{k}_i^t < N$, agents use a statistics $\Re$. $\Re$ is computed over individual payoffs an agent i would earn if individual strategies within her network V_i^t were fixed, while individual strategies of agents $j \in I - \{V_i^t\}$ can change freely in $\{-1, +1\}^{N-k_i^t-1}$. Let:

$$I = \overline{V}_i^t \cup W_i^t, \overline{V}_i^t \cap W_i^t = \emptyset,$$

where $\overline{V}_i^t = \{i, j_1, \ldots, j_{K_i^t}\}$ and $W_i^t = \{h_1, \ldots, h_{N-K_i^t-1}\}$. Define $P_i^t \subseteq \{-1, +1\}^N$ as the set of all possible configurations $\Omega^t = (s_i^t)_{i \in I} \in \{-1, +1\}^N$ for which $\Omega^t(\overline{V}_i^t)$ are kept constant while $\Omega^t(W_i^t)$ vary freely. We assume that if $k_i^t = 0, \ldots, N-1$ then:

$$\pi_i^t(\Omega^t(\overline{V}_i^t)) = \Re(\pi_i^t(\Omega^t), \Omega^t \in P - i^t),$$

i.e. $\pi_i^t(\Omega^t(\overline{V}_i^t))$ are computed by employing statistics $\Re$ over all possible configurations where only the strategies of agents outside $\overline{V}_i^t$ can take all admissible values.

Agents use one out of four criteria to evaluate such payoffs: (i) $\Re = \text{MAX}$; (ii) $\Re = \text{MIN}$; (iii) $\Re = \text{MEAN}$; (iv) $\Re = \text{RND}$. In the last case, agent i computes her payoff by picking a payoff at random out of the admissible ones.

At time $t = 0$, a strategy configuration $\Omega^0 \in \{-1, +1\}^N$ and a graph G^0 over I are drawn at random. At any time period $t \geq 1$, agents update their networks, given Ω^t, then update their strategies given the just updated graph. Given (Ω_t, G_t), any two agents i and j, either connected in G_t or not, are picked up at random from I. They evaluate current payoffs as:

$$w_i^t = \pi_i^t(\Omega_t(\overline{V}_i^t)),$$
$$w_j^t = \pi_j^t(\Omega_t(\overline{V}_j^t)).$$

If i and j are not connected ($ij \notin G_t$), define $\widetilde{V}_i^t = \overline{V}_i^t \cup \{j\}$ and $\widetilde{V}_j^t = \overline{V}_j^t \cup \{i\}$. If i and j are already connected ($ij \in G_t$), define $\widetilde{V}_i^t = \overline{V}_i^t - \{j\}$ and $\widetilde{V}_j^t = \overline{V}_j^t - \{i\}$. Accordingly, define payoffs under addition or deletion:

$$\widetilde{w}_i^t = \pi_i^t(\Omega_t(\widetilde{V}_i^t)),$$
$$\widetilde{w}_j^t = \pi_j^t(\Omega_t(\widetilde{V}_j^t)).$$

Agents decide bilaterally to add or delete the link by comparing current payoffs before and after the change and by selecting the network with higher payoff. We consider two alternative tie-breaking rules:

1. Add or delete the link if and only if both agents are strictly better off under the change, i.e., if and only if $\widetilde{w}_i^t > w_i^t$ and $\widetilde{w}_j^t > w_j^t$. This tie-breaking rule is called *without neutrality*.
2. Add or delete the link if and only if no agent is strictly worse off under the change, if and only if $\widetilde{w}_i^t \geq w_i^t$ and $\widetilde{w}_j^t \geq w_j^t$. This tie-breaking rule is called *with neutrality*.

Network updating is therefore defined as follows:

$$G^{t+1} = \begin{cases} G^{t+1} = G^t \cup \{ij\} & \text{if the link is added} \\ G^{t+1} = G^t - \{ij\} & \text{if the link is deleted} \\ G^{t+1} = G^t & \text{otherwise} \end{cases}$$

After network updating, strategy updating takes place given G_{t+1}. An agent i is drawn at random from I. Given $(s_i^t, V_{I,t+1})$ and her current payoff

$\omega_i{}^t = \pi_i^t\left(s_i^t, \Omega^t\left(\overline{V}_i^{t+1}\right)\right)$, she will switch to $-s_i^t$ at the beginning of period $t+1$ if and only if:

$$\pi_i^t\left(-s_i^t, \Omega^t\left(\overline{V}_i^{t+1}\right)\right) > w_i^t,$$

We now provide results about the long-run behavior of the process governing the evolution of the pair $\{\Omega^t, G^t\}$.

4. SIMULATING THE SYSTEM

Unless otherwise specified, all results refer to Montecarlo averages across 50 independent populations. Each population includes 15 agents, each using an independently drawn payoff matrix containing 2^{15} entries, uniformly distributed in the [0, 1] interval, for a total of $2^{15} \times 15$ random values. Such payoff matrices are used for all populations in the same simulation, so that we can compare the results from different populations. Given distributional assumptions on payoff values, our results do not change qualitatively with different payoff landscapes across independent sets of simulations.

Before presenting our results, a brief comment on payoff specification is in order. Our agents are assigned a payoff that depends on the strategies of all other agents, besides their own strategy. The network structure determines both the amount of missing information (i.e., the strategies of the unconnected agents) and the amount of information that is perfectly known to any individual (i.e., the strategies of connected agents). Evaluation criteria determine how missing information is elaborated.

We chose to use random payoffs uniformly distributed to avoid any possible bias in the results due to a specific exogenous structure of the payoff distribution. In fact, we were interested in observing the emergence of endogenous networking structures caused by the use of given evaluation rules. We did not want the results to depend on a specific payoff distribution, however realistic this might have been.

In this section we start simulating agents with "frozen" networks: agents are allowed only to change their strategies but not their links, fixed in the beginning of the simulation run. These exercises will tell how a given structure influences the agent's payoff. We will present the results from the full model, where strategy and network updating take place given different evaluation rules.

4.1 Fixed Networks

In order to appreciate how the connectivity of the network influences payoffs, we first simulate agents who can modify their strategies but not their links. We generated 100 random networks kept fixed in the rest of the simulation. We let the probability that, taken two agents, a link is established between them to range from 0.01 (almost totally unconnected network) to 1.00 (fully connected one).

Simulations show that, if the average total number of links is sufficiently large, then no steady-state is ever reached and all populations continue to explore the payoff landscape. Figure 1 presents the main results with the MEAN evaluation statistics. The expected value of payoff is always 0.5, but the variance increases with the total number of links. Agents with no or few links have always payoffs very close to 0.5, because this value is computed as the average over a large number of random values. As the total number of links increases, payoffs become more and more scattered on the interval [0, 1], because the MEAN statistics is computed on smaller and smaller sets of uniformly distributed random values.

Simulations using the MIN and MAX payoff rules produce similar results, with payoffs for unconnected agents concentrated around 0 and 1 for the minimum and the maximum respectively. In turn, simulations with RND payoff rule exhibit payoffs ranging over the entire interval [0, 1], independently of the total number of links.

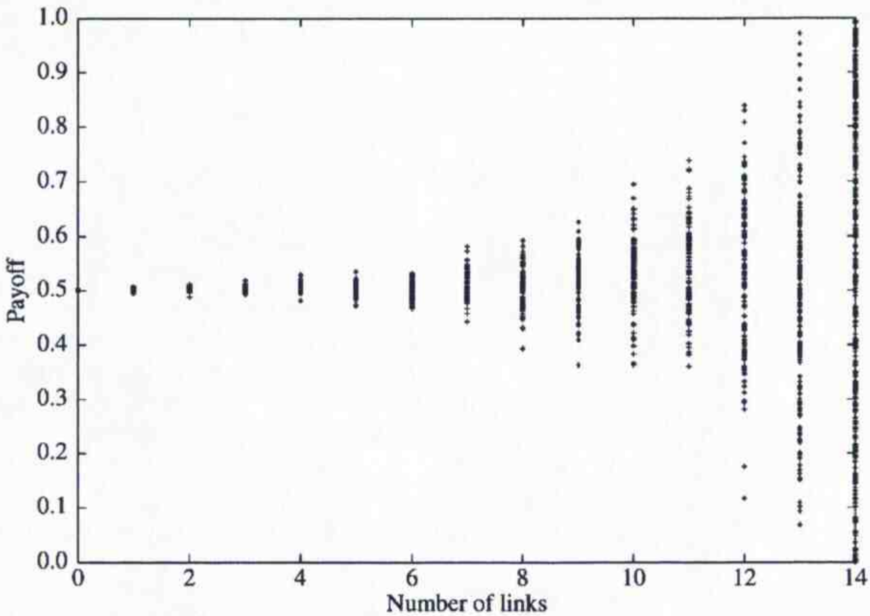

FIGURE 1 Fixed networks with players employing MEAN evaluation rules. Scatter-plot of individual payoffs (y-axis) vs. individual number of links (x-axis). Values at T = 10000. 50 populations, 15 agents each.

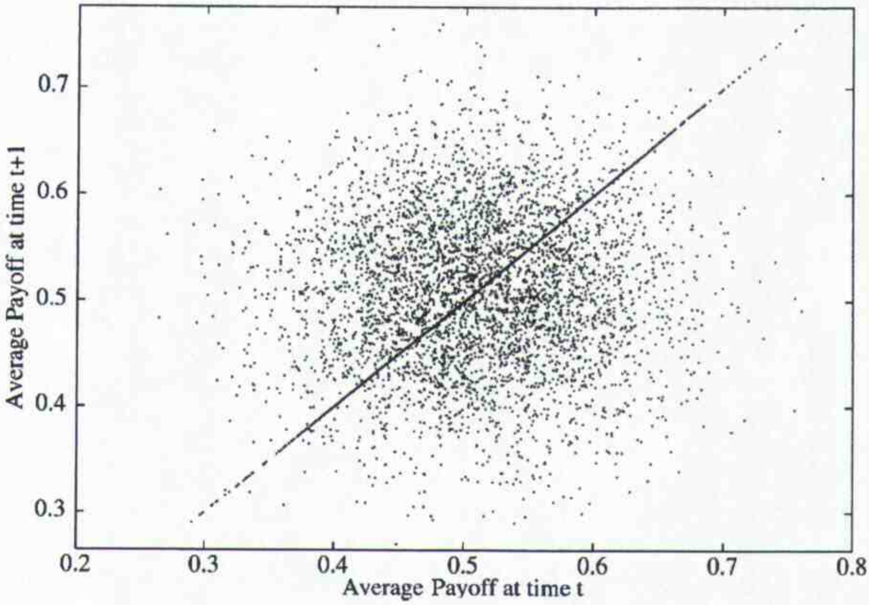

FIGURE 2 Fixed networks with players employing MEAN evaluation rules. Scatter-plot of population average payoffs at time t + 1 (y-axis) vs. population average payoffs at time t (x-axis). Values after 10000 time steps. 50 populations, 15 fully connected agents each.

Beside the absolute values of payoffs, the total number of links an agent holds also affects how many updates improving the population average payoff are performed. In fact, when agents with fewer links switch their strategies, they affect only the few agents to which they are connected. Changes in strategy have therefore little effect on the population average payoff. Conversely, populations of highly connected agents have highly volatile average payoffs, because any strategy mutation alters the payoff of many agents. In Figure 2 we report the phase diagram for the population average payoff when agents are fully connected[14]. The graph plots the population average payoff at time $t + 1$ as a function of the same variable at time t. As agents are change-averse with respect to action-updating, points on the diagonal indicate all cases in which no strategy switch has taken place. When a switch happens (off-diagonal), the distribution of points shows no significant correlation.

[14] The graph refers to a simulation with the MEAN payoff rule, although practically the same result is obtained also for any other type of payoff rule.

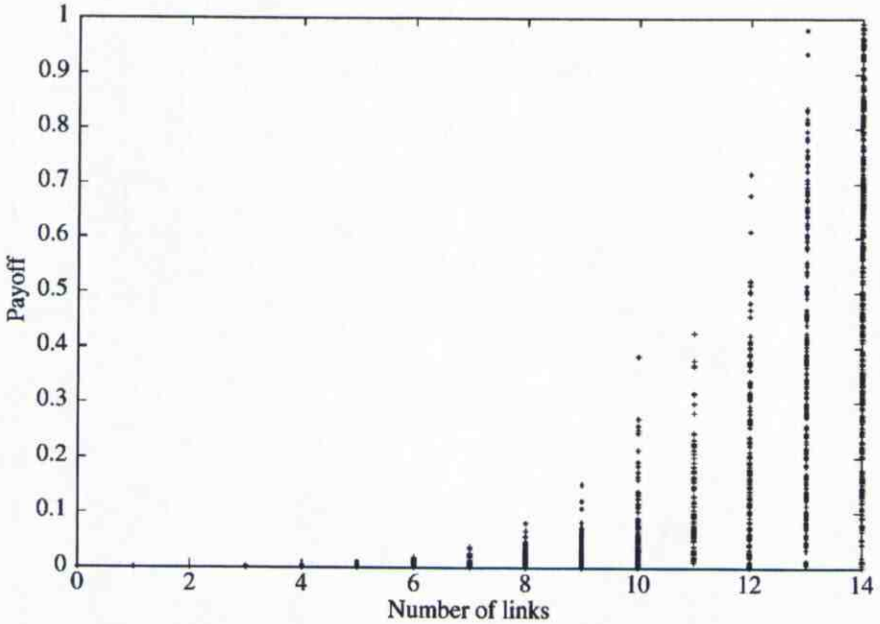

FIGURE 3 Fixed networks with players employing MIN evaluation rules. Scatter-plot of individual payoffs (y-axis) vs. individual number of links (x-axis). Values at T = 10000. 50 populations, 15 agents each.

The MIN and MAX evaluation statistics show similar patterns. The expected payoff distribution for agents holding few links is concentrated around 0 for MIN or around 1 for MAX, as Figures 3 and 4 show. Therefore, when static networks are assumed, the long-run relationship between the distribution of individual payoffs and the total number of links held by the agents is driven for any payoff rule by the trade-off between average and variance induced by the chosen criterion.

4.2 Endogenous Networks

4.2.1 Mean and RND Evaluation Rules

Agents now change adaptively not only their strategies but also the set of neighbors with whom they interact by adding and deleting connections. We use 50 populations made of 15 agents, and the same payoff matrices for all populations.

We start again by considering the MEAN evaluation statistics and we initialize our simulations with totally unconnected networks and random initial strategies.

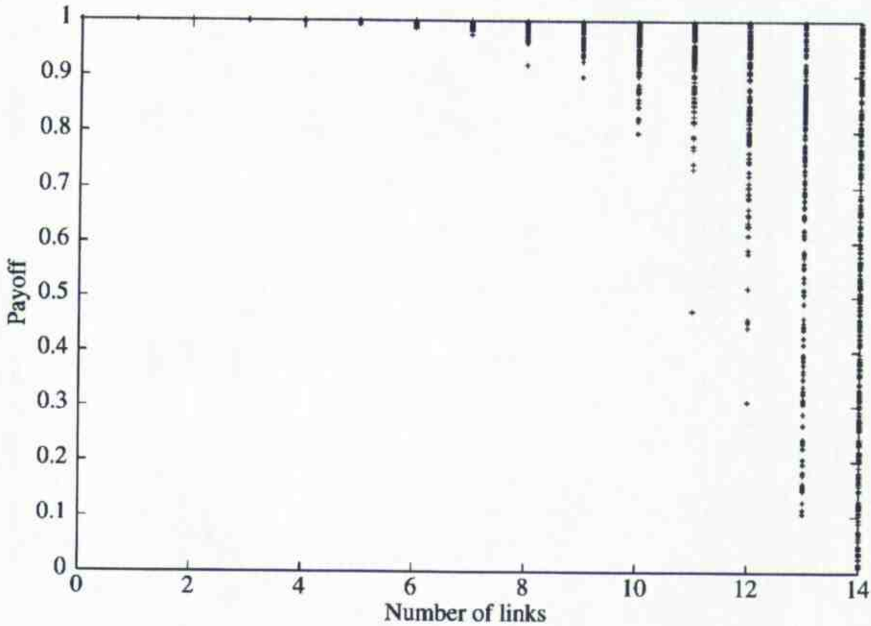

FIGURE 4 Fixed networks with players employing MAX evaluation rules. Scatterplot of individual payoffs (y-axis) vs. individual number of links (x-axis). Values at T = 10000. 50 populations, 15 agents each.

Irrespective of neutrality, the system displays many steady states. Figure 5, showing the average number of links for each population, suggests that all populations display a similar dynamic pattern characterized by distinct phases, though the duration of these phases may vary. At the outset, all agents in the population produce successful updating both on strategies and on graphs, by exploring different strategies and networks. They stop updating their strategies but still modify their networks, adding or removing links. Finally, network updating stops, with the agents making no further modifications. Payoffs and links for the 15 agents in the steady-state configuration widely differ within each simulation and across simulations. The total number of links can range from 1 to 13 and payoffs from 0.1 to 0.98. Population averages range between 6.7 to 11.1 for the total number of links (Figure 5), and between 0.48 to 0.69 for payoffs. These various stable points differ from one population to another, and the system displays many "local optima" corresponding to lock-in points for updating rules.

On Figure 6 we report the scatter plot of agents' payoffs as a function of the number of their links. Compared to frozen networks (see Figure 1), endogenous network updating allows agents with wider neighborhood sets

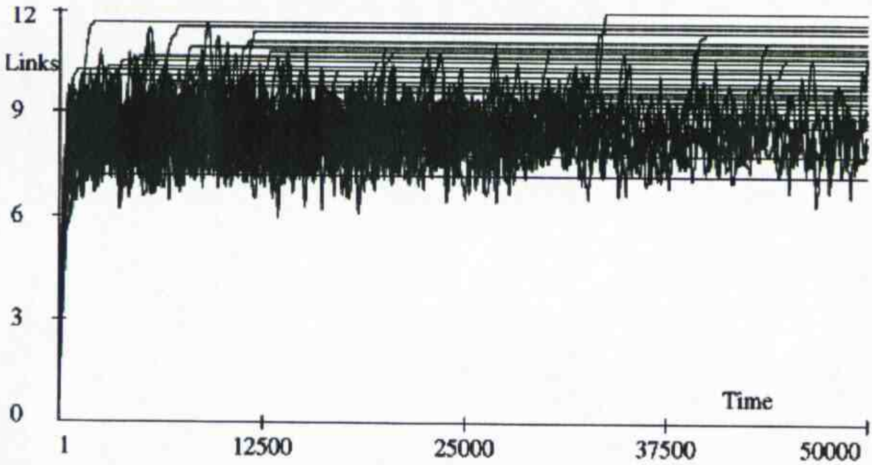

FIGURE 5 Endogenous networks with players employing MEAN evaluation rules. Time series of population average number of links. 50 populations, 15 agents each.

to persistently reach higher payoffs: agents adapt so as to concentrate on the highest portion of the payoff space.

These results are not affected by the initial conditions. The dynamic behavior of a system based on a RND evaluation rule is instead similar to that displayed by static networks. Indeed, a RND rule fails to exhibit a positive relationship between the total number of links and average payoffs. Moreover, strategy and network go on being updated indefinitely and a stable structure is never achieved. This result is consistent with the observation that agents who use this payoff evaluation rule have no reason to prefer few to many links, because the payoff is always obtained randomly.

4.2.2 MIN Evaluation Rule

When agents use the MIN payoff evaluation rule, they compute their payoff as the minimum of the values in their payoff matrix corresponding to all combinations of strategies used by the agents with whom there is no direct interaction. This payoff rule is played by "pessimistic" agents, in that they maximize the payoff in the worst possible case, considering these as the unobservable strategies from the non-linked agents.

Contrary to the previous cases, results heavily depend upon the acceptance of neutral network updating (i.e., whether agents add or remove a link although their payoff remains unchanged). Increasing or decreasing the total number of links held by an agent respectively narrows or widens the set of payoffs from which the minimum is taken. It often happens that adding or deleting links leaves the minimum value of such a set unchanged. These network changes are "neutral" with respect to the

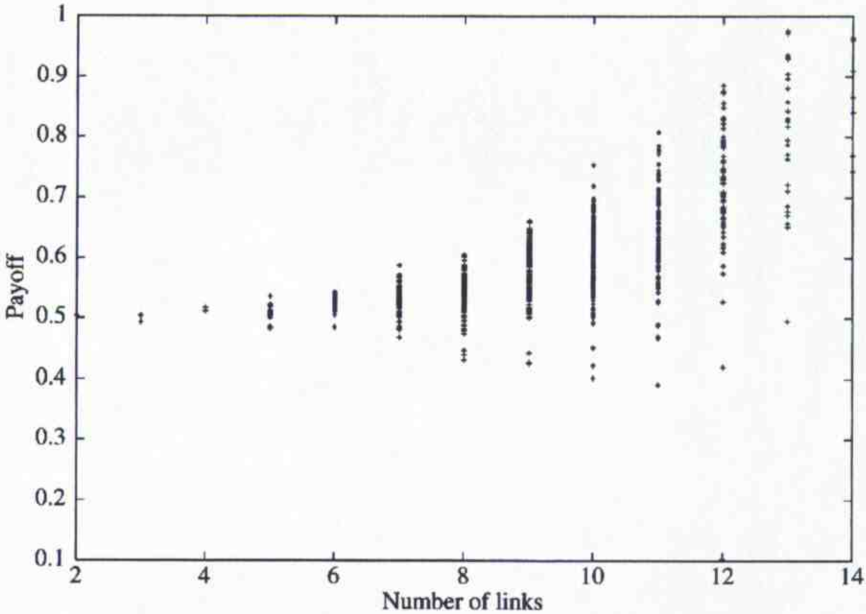

FIGURE 6 Endogenous networks with players employing MEAN evaluation rules. Scatterplot of individual payoffs (y-axis) vs. individual number of links (x-axis). Values at T = 50000. 50 populations, 15 agents each.

payoff evaluation. Whether such neutral changes are acceptable or not is thus important to the dynamic of the network structure.

Figures 7 and 8 present the time series of the average number of links for the 50 simulated populations and the plot of average payoffs against total number of links respectively, assuming agents accept a payoff-neutral change. Independently of the connectedness of the initial graph, the structure of the network and the choices of strategy never converge to a steady-state. The long-run average number of links fluctuates between about 10 and 13. A positive correlation between average payoffs and average number of links, although weak, still comes out (Figure 8).

If on the contrary agents do not accept neutral network updating (they are change-averse), simulations show convergence to multiple steady networks, but strategies, and therefore payoffs, never settle. In fact, in this case a link between any two agents can never be deleted, because the minimum payoff cannot increase[15], new links can always be formed. The

[15]This is the case because, if an agent removes a link, the pool of payoffs across which the minimum is computed strictly contains the payoff set associated to the network with that link still in place.

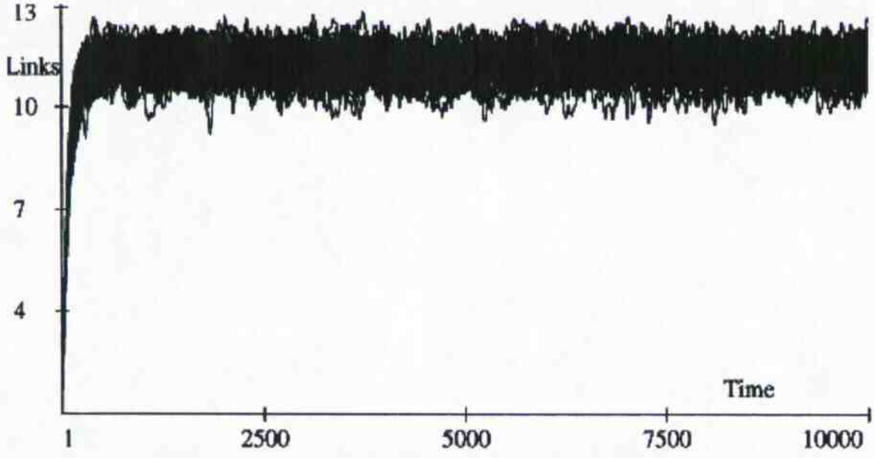

FIGURE 7 Endogenous networks with players employing MIN evaluation rules and accepting payoff-neutral network changes. Time series of population average number of links. 50 populations, 15 agents each.

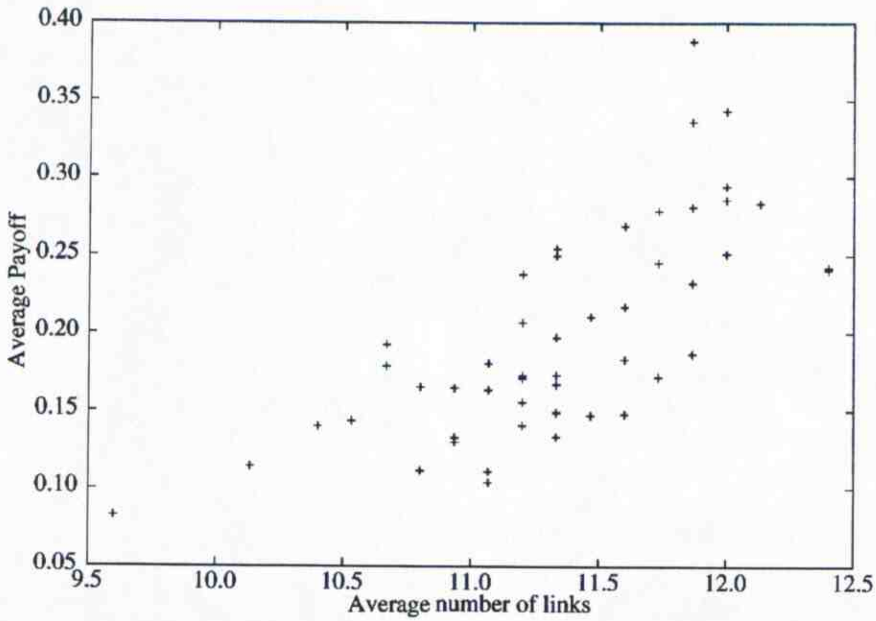

FIGURE 8 Endogenous networks with players employing MIN evaluation rules and accepting payoff-neutral network changes. Scatter plot of average payoff (y-axis) as a function of the average number of links (x-axis). Values at T = 50000. 50 populations, 15 agents each.

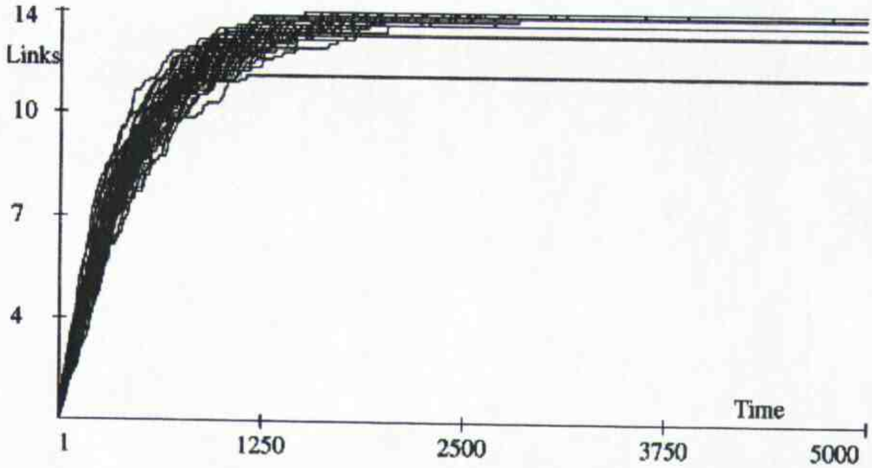

FIGURE 9 Endogenous networks with players employing MIN evaluation rules but NOT accepting payoff-neutral network changes. Time series of population average number of links. 50 populations, 15 agents each.

network becomes highly connected and almost all agents maintain a total number of links close to the maximum (14), as illustrated on Figures 7 and 9.

When agents accept neutral updates, further changes in the network structure are very likely to occur, but this in turn changes individual payoffs importantly. The system never climbs an optimum, even local. Conversely, if agents are change-averse with network updating, they typically get stuck in one of the many local maxima. Strategy updating never stops, because changes of an agent's strategy induce considerable changes in the other agents' payoffs, and in general offer opportunities for some of them to change, in turn, their strategies.

Our result means that pessimistic agents, who base actions on worst-case considerations, tend to increase more and more their span of control in order to reduce the risk of unexpected worst cases. However, this behavior collectively generates instability, especially when payoff-neutral network updating is always accepted, and when players can explore a larger portion of the payoff landscape.

4.2.3 Max Evaluation Rule

With the MAX payoff evaluation criterion, agents adopt a sort of optimistic criterion, as they base their decisions upon the best payoffs obtainable from the behavior of agents outside their own neighborhood. As with the MIN criterion, results depend upon the acceptance of neutral network updating.

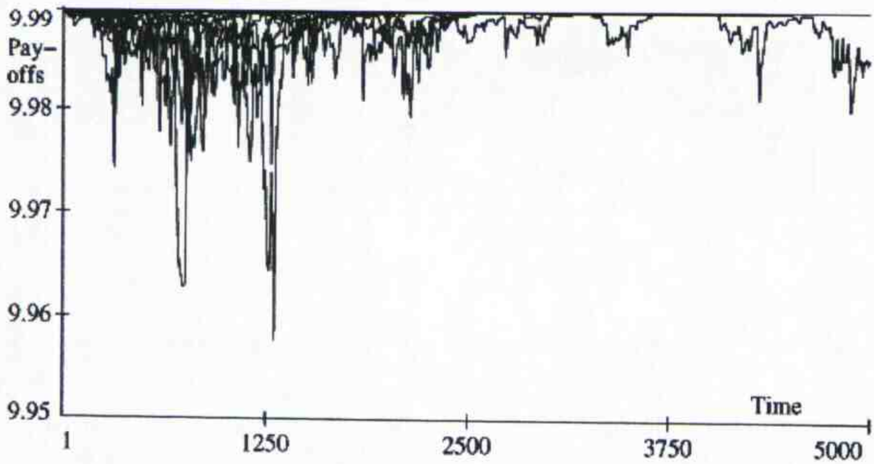

FIGURE 10 Endogenous networks with players employing MAX evaluation rules and accepting payoff-neutral network changes. Time series of population average payoffs (multiplied by 10). 50 populations, 15 agents each.

In case of acceptance, agents tend to develop networks with very few connections, because adding a link can never strictly increase the maximum payoff[16]. On the contrary, link deletion can increase the maximum payoff and then can be accepted. As the population tends to produce scarcely connected networks, agents can tune their actions in order to reach the global optimum payoff. Figure 10 reports the average payoff for the 50 populations and shows that each population always converges to the same highest payoff, very close to 1[17]. If one disaggregates the average payoff in each population, all populations converge to the same strategy profile and the same maximum individual payoffs, for a given payoff matrix and from any initial condition. Once the optimal strategy profile is reached, some payoff neutral network changes are still possible and therefore the network structure never stabilizes. However, all network structures over which the population keeps cycling indefinitely are payoff equivalent.

Results differ if agents cannot make strategy or network changes unless for a strictly higher payoff (*non neutrality*). If the initial network is unconnected, no link is ever established as none of them is strictly payoff-increasing. If the

[16]The new payoff is computed as the maximum of a set which is strictly contained in the one associated to the network containing that link. Only if the maximum is still contained in the subset can the link addition be accepted under *neutrality*.

[17]One population fails to converge, though it is still a matter of time. Sooner or later all agents discard almost all the links and individuate the state producing the maximum payoff, though this may need some time.

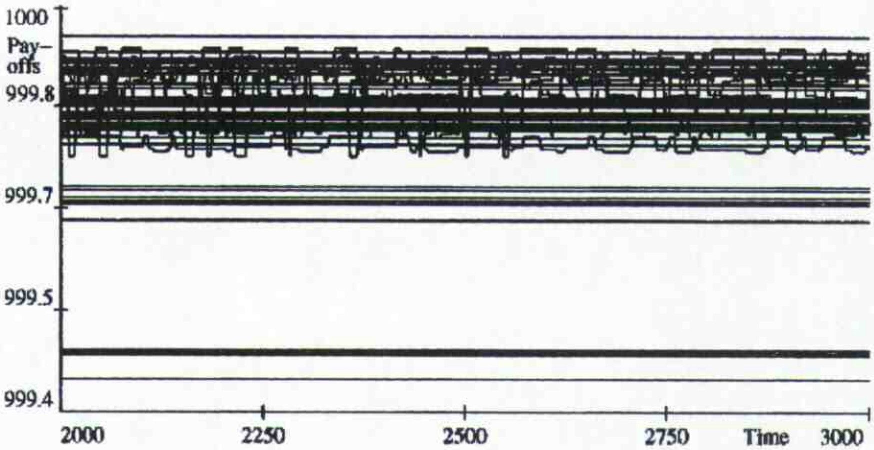

FIGURE 11 Endogenous networks with players employing MAX evaluation rules but NOT accepting payoff-neutral network changes. Time series ($t = 2000, \ldots, 3000$) of population average payoffs (multiplied by 1000). 50 populations, 15 agents each.

initial network is fully or highly connected, denying payoff neutral changes deters agents from climbing the entire payoff landscape and they end up locked into local optima. The deletion of a link requires that both agents in a pair are strictly better off after deletion: if one of them is indifferent, the link cannot be removed and the other agent is locked into a suboptimal set of neighbors. Figure 11 plots the average payoffs for the 50 populations given the same payoff matrix, starting with fully connected agents, and not accepting neutral changes. Populations lock into different local optima depending on initial conditions[18]. Therefore, allowing for changes in the payoff neutral network leads to many steady-states, in both networks and actions, characterized by few links and different levels of average payoff.

5. CONCLUSIONS

Population learning in dynamic economies has been traditionally studied in simplified settings where all agents play the same bilateral stage-game against any opponent and stage-game payoffs reflect very simple strategic situations.

We have investigated dynamic population games over rugged landscapes, where agents face a strong uncertainty about expected payoffs from bilateral interactions. We suggested to model payoff landscapes

[18]Average payoffs are very high because the MAX rule is used.

through random population games where the population adapts, over both strategies and network structures, using deterministic, myopic, best reply rules. The key assumption concerns how agents evaluate payoffs associated to networks which imply local interactions. We explored settings where players use very simple statistics computed on the distributions of payoffs associated to all combinations of possible actions of agents outside the interaction set.

Computer simulations showed that: (1) endogenous networks imply higher average payoffs as compared to exogenous static networks; (2) the statistics employed to evaluate payoffs strongly affect the dynamic properties of the system, and in particular its convergence to a unique or multiple steady state; and (3) for the MIN and MAX statistics, the likelihood of efficient population learning depends on whether agents are change-averse or not in discriminating between options yielding the same expected payoff.

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