Digital Signal Processing for Compensating Fiber Nonlinearities

Domenico Marsella, Marco Secondini, Enrico Forestieri
Institute of Communication, Information, and Perception Technologies
Scuola Superiore Sant'Anna
Pisa, Italy

***Abstract*—Successful compensation of nonlinear distortions due to fiber Kerr nonlinearities relies on the availability of an accurate channel model. Some models obtained by approximate solutions of the nonlinear Schrödinger equation and the backpropagation method are taken into account. It turns out that backpropagation is not the optimal processing technique and in some cases is outperformed by simpler processing techniques.**

***Keywords*—fiber nonlinearities; backpropagation; maximum likelihood detection**

I. Introduction

The nonlinear Schrödinger equation (NLSE) describes the propagation of the optical field complex envelope $v(z,t)$ in a single-mode fiber. Accounting for group velocity dispersion (GVD), self-phase modulation (SPM), and loss, in a time frame moving with the signal group velocity, the NLSE can be written as [1]

$$\frac{\partial u}{\partial z} = j\frac{\beta_2}{2}\frac{\partial^2 u}{\partial t^2} - j\gamma e^{-\alpha z}|u|^2 u \tag{1}$$

where $u(z,t) = \exp(\alpha z/2)\, v(z,t)$ is the normalized complex envelope, β_2 is the GVD parameter, γ is the Kerr nonlinear coefficient, and α is the power attenuation constant. In general, the NLSE does not admit an analytical solution when both β_2 and γ are different from zero. Only when $\alpha = 0$ the exact solution can be obtained by the inverse scattering method in the form of classical solitons. If the attenuation cannot be neglected, the NLSE can be approximately solved either numerically, through the split-step Fourier method (SSFM) [2], or analytically, through the Volterra series approach [3]. These two different approaches naturally lead to two different strategies for the compensation of nonlinear effects: nonlinear equalization based on Volterra kernels [4] and channel inversion by digital backpropagation (BP) [5]. However, when applied to typical long-haul optical links, the first approach requires too many Volterra kernels to obtain a good approximation of the output field, making the corresponding nonlinear equalizer too complex. Better results can be obtained with BP. However, this technique also requires a complex structure and, as discussed in the following, is not the optimal detection strategy in a real scenario.

II. Perturbation Methods

An alternative approach to approximately solve the NLSE is based on perturbation methods.

A. Regular Perturbation

The regular-perturbation (RP) method is a classical method used for solving nonlinear differential equations [6]. The (normalized) optical field complex envelope $u(z,t)$ is expanded in power series of the nonlinear coefficient γ. The first term $u_0(z,t)$ of the series is the linear solution of the NLSE ($\gamma = 0$), while the other terms describe the deviation from the linear solution due to nonlinear effects. Truncating the series after the first two terms, we obtain

$$u(z,t) \approx u_0(z,t) + \gamma u_1(z,t) \tag{2}$$

with

$$u_1 = -j\int_0^z (|u_0|^2 u_0) \otimes h(z-\zeta,t)e^{-\alpha\zeta}d\zeta \tag{3}$$

where $\otimes$ is the time convolution operator and $h(z,t)$ is the linear impulse response of a fiber of length z. Remarkably, the RP method has been shown to be equivalent to the Volterra series approach [6].

B. Logarithmic Perturbation

The logarithmic perturbation (LP) method consists in expanding the log of the complex envelope $u(z,t)$ in power series of γ. By retaining only the first term of the expansion, $u(z,t)$ can be approximated as [1]

$$u(z,t) \approx u_0(z,t)\exp\big(\gamma\psi_1(z,t)\big) \tag{4}$$

where $u_0(z,t)$ is again the linear solution of the NLSE and $\psi_1(z,t) = u_1(z,t)/u_0(z,t)$, $u_1(z,t)$ being as in (3).

C. Combined Regular-Logarithmic Perturbation

In order to overcome the difficulty that the linear solution of the NLSE appears at the denominator of $\psi_1(z,t)$ in (4), a combined regular-logarithmic perturbation (CRLP) was developed [7]. By extending the CRLP method to modulated signals and adopting a CRLP expansion in γ, the output signal can be approximated as

$$u(z,t) \approx \big(u_0(z,t) + \gamma\upsilon(z,t)\big)\exp\big(-j\gamma\varphi(z,t)\big) \tag{5}$$

where $u_0(z,t)$ is again the linear solution of the NLSE, while

$$\varphi(z,t) = \int_0^z |u_0(\zeta,t)|^2 e^{-\alpha\zeta}d\zeta \tag{6}$$

$$\upsilon(z,t) = \int_0^z f(\zeta,t) \otimes h(z-\zeta,t)d\zeta \tag{7}$$

where

$$f(z,t) = \frac{\beta_2}{2}\Big(u_0\frac{\partial^2\varphi}{\partial t^2} + 2\frac{\partial u_0}{\partial t}\frac{\partial \varphi}{\partial t}\Big). \tag{8}$$

D. Comparison

In order to compare the accuracy of the various approximations, we assume that the "true" solution $u(z,t)$ can be obtained by the SSFM with a sufficiently small step size and adopt a normalized square deviation (NSD) defined as

$$\mathrm{NSD} = \int |u(z,t) - u_P(z,t)|^2\, dt \Big/ \int |u(z,t)|^2\, dt \qquad (9)$$

where $u_P(z,t)$ is one of the previous approximations and the integrals extend over the whole transmission period. The NSD was computed for two different configurations of a typical 10×130 km terrestrial link, whose spans employ a fiber with a dispersion coefficient of 4.4 ps/nm/km, a nonlinear coefficient of $1.3\,\mathrm{W}^{-1}\mathrm{km}^{-1}$, and attenuation of 0.23 dB/km. The NSD reported in Fig. 1(a) refers to a dispersion compensated link, where a compensating fiber (DCF) with a dispersion coefficient of −100 ps/nm/km, nonlinear coefficient of $6.5\,\mathrm{W}^{-1}\mathrm{km}^{-1}$ and attenuation of 0.57 dB/km is sandwiched in a dual stage amplifier with noise figures of 7.5 dB and 6.5 dB. The DCF length is chosen to exactly compensate for the accumulated dispersion. Instead, Fig 1(b) reports the NSD for the case of no in-line dispersion compensation. In both cases, the gain of the dual-stage amplifiers recovers the span loss, so that the power launched in each fiber is equal to the transmitted power. The input field is a 50 Gb/s QPSK-NRZ signal, generated by a nested Mach-Zehnder modulator with 20 GHz bandwidth, filtered by a 4^{th}-order Gaussian bandpass filter with 45 GHz bandwidth and modulated by a quaternary de Bruijn sequence of length2^{12}. The received signal is then filtered by a 4^{th}-order Gaussian bandpass filter with 40 GHz bandwidth. In either considered cases, the LP method turns out to be the best approximation. If we consider acceptable an NSD $< 10^{-2}$, the LP method allows for a launch power of $6 \div 7$ dBm.

III. Nonlinear Channel Models

The methods examined in the previous sections can be used as a hint to devise an optimum receiver according to the maximum a posteriori (MAP) criterion, given that they suggest a model for the nonlinear fiber optic channel. In other words, we do not try to devise a nonlinear equalizer based on the approximate solutions of the NLSE, but rather use them to infer a channel model for applying MAP strategies.

A. Backpropagation

If all the amplified spontaneous emission (ASE) noise were injected only at the input of the link, the BP method could exactly invert the NLSE and the channel would revert to the additive white Gaussian noise (AWGN) case. In this case, according to the MAP criterion, the optimum receiver would be a matched filter followed by a symbol-by-symbol (SxS) detector. In practical cases, however, ASE noise is injected along the link by in-line amplifiers, such that BP followed by SxS detection is not optimum and a better (yet, sub-optimum) performance is obtained by a partial BP on a reduced number of spans.

B. Cartesian Gaussian MLSD

In a linear propagation regime, the output signal can be written as $u_0(z,t) = s_0(z,t) + n_0(z,t)$, where $s_0(z,t)$ is the signal component, affected by dispersion, and $n_0(z,t)$ is the

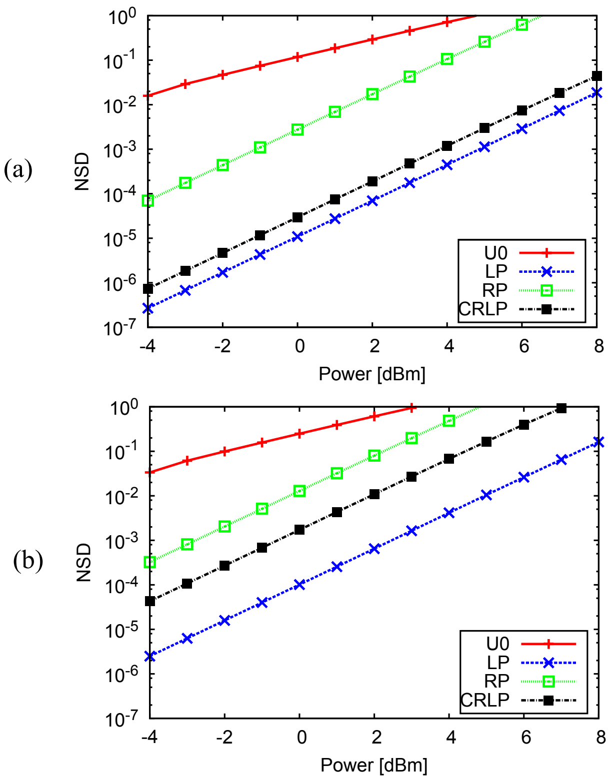

Fig. 1. Normalized square deviation for (a) dispersion-compensated and (b) dispersion-uncompensated link. The curve labeled U0 corresponds to the linear solution of the NLSE.

AWGN due to optical amplifiers. According to the RP method, in the presence of nonlinear effects, the output signal is also affected by an additive perturbation term, $\gamma u_1(z,t)$, that depends both on $s_0(z,t)$ and $n_0(z,t)$ through (3). Retaining only terms that are linear in $n_0(z,t)$, as done for instance in [7], the output signal is affected by nonlinear intersymbol interference (ISI) and colored Gaussian noise, such that the in-phase and quadrature components of the output samples, conditional on the transmitted symbols, can be modeled as correlated Gaussian variables. Thus, the MAP strategy takes the form of maximum likelihood sequence detection (MLSD), implemented by a Viterbi algorithm (VA) based on Cartesian Gaussian (CG) metrics [8].

C. Polar Gaussian MLSD

Also the CRLP terms in (5), $\upsilon(z,t)$ and $\varphi(z,t)$, can be linearized with respect to the noise term $n_0(z,t)$. In this case, the output signal is still affected by nonlinear ISI, but its distribution is better approximated by assuming that the amplitude and phase of the output signal samples are correlated Gaussian variables, rather than the in-phase and quadrature components. Consequently, the MAP strategy could be implemented by a VA based on polar Gaussian (PG) metrics [8].

D. Comparison

Fig. 2 compares the performance of all the detection strategies in terms of bit error rate (BER) versus launch power for the same dispersion-compensated link introduced in the previous section. Ideal BP is performed on the op-

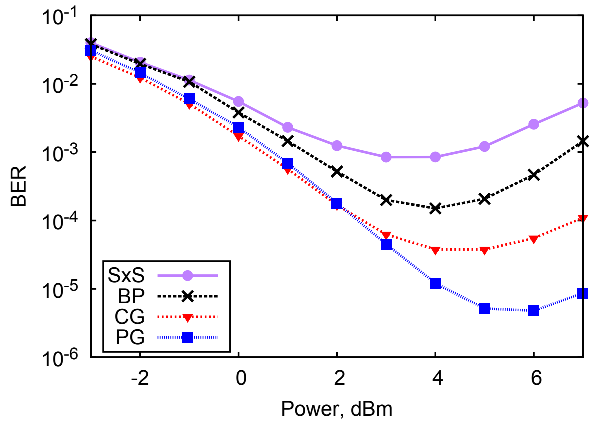

Fig. 2. BER versus launch power for the described detection strategies.

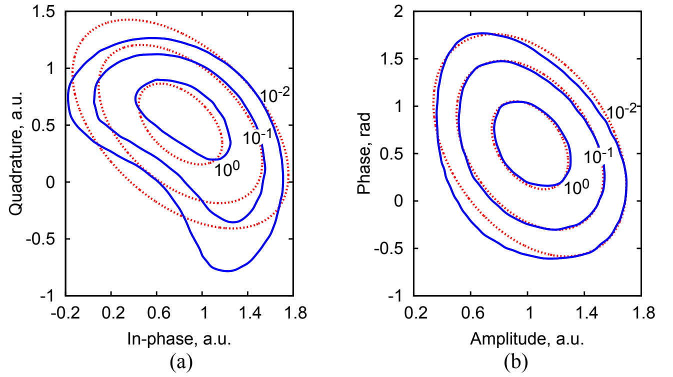

Fig. 3. Joint distribution (contour lines) of (a) in-phase and quadrature and (b) amplitude and phase components. Solid lines refer to simulations, dashed lines to Gaussian fitting.

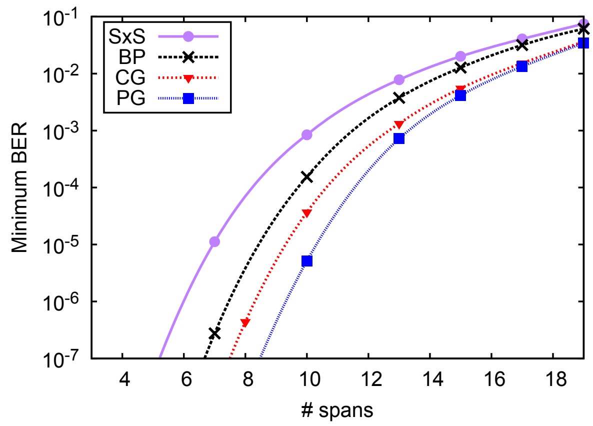

Fig. 4. Minimum BER versus link length (130 km per span) for the described detection strategies.

timum number of spans (five in this case). The VA with CG or PG metrics is implemented with 16 states (nonlinear ISI limited to 2 symbols). For each state and symbol, the required conditional expectation and covariance matrix are estimated and stored in a look-up table using a suitable training sequence and assuming that samples spaced more than a symbol time are uncorrelated. Since low dispersion fiber and in-line compensation are considered, nonlinear ISI is limited to a few symbols and can be handled by the 16-state VA, while signal-noise interaction is highly detrimental and gives rise to parametric gain and nonlinear phase noise. Therefore, BP, that is beneficial only against nonlinear ISI, brings some improvement with respect to SxS, but has a lower performance compared to VA detectors, which can cope also with parametric gain. In addition, the PG metric better accounts for nonlinear phase noise and outperforms the CG metric and the BP algorithm both in terms of minimum BER and of required power for a given BER. This behavior can be better understood by looking at the distribution of the received sample conditional on the transmitted sequence for a launch power of 6 dBm (minimum BER for the PG detector). Fig. 3(a) compares the joint distribution of the in-phase and quadrature components of the signal, obtained by Monte Carlo simulations, with a bi-variate Gaussian distribution with same expectation and covariance matrix (i.e., the distribution adopted by the CG metric). Fig. 3(b) reports a similar comparison but considering the amplitude and phase of the received sample (whose distribution is assumed Gaussian by the PG metric). It is apparent the higher accuracy of the PG metric, while the CG metric can only provide a rough estimate of the parametric gain effect. A comparison among the detection strategies for different length of the link is finally reported in Fig. 4, where only the minimum of each BER curve is reported as a function of the number of spans. The VA with PG metric always gives the minimum BER (for a given link length) or the maximum reach (for a fixed BER threshold).

IV. Conclusions

We have presented different channel models for optical fibers affected by Kerr nonlinearity and derived the related optimum detection strategies. For dispersion-compensated links deploying low-dispersion fibers, the best performance is obtained by a 16-state Viterbi detector with polar Gaussian metrics, while a worse performance (both in terms of maximum reach or minimum BER) is obtained by digital backpropagation, despite its higher complexity. This work was supported in part by Ericsson and by MIUR under the FIRB project COTONE.

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